R-16

Code: 4308/19

Faculty of Science

B.Sc (Mathematics) II-Year, CBCS –IV Semester Examinations, May/June 2019

PAPER: ALGEBRA

Time: 3 Hours Max Marks: 80

Section-A

I. Answer any FIVE of the following questions. (5x4=20 Marks)

1. Find the inverse of the element $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in GL(2,Z_{11}) with respect to matrix multiplication.
2. Write the permutation (12)(13)(23)(142) as product of disjoint cycles.
3. Show that if a group G has a unique subgroup H of some finite order, then H is normal in G.
4. If $\phi : G \to \overline{G}$ is a group homomorphism, then show that Ker ϕ is a normal subgroup of G.
5. Find all elements of $\frac{Z(i)}{<2-i>}$
6. Prove that the only ideals of a field F are {0} and F itself.
7. Determine all ring homomorphisms from $Z_{20} \to Z_{30}$.
8. If $f(x)=4x^3+2x^2+x+3, g(x)=3x^4+3x^3+3x^2+x+4$ are polynomials over the ring $Z_5[x]$ then find $f(x).g(x)$.

Section-B

II. Answer the following questions. (4x15=60 Marks)

9. (a) (i) If H is a nonempty finite subset of a group G and closed under the operation of G,then show that H is subgroup of G.
   (ii) Show that the center Z(G) of a group G is a subgroup of G.
   (OR)
   (b) State and prove the fundamental theorem of cyclic groups.

10. (a) (i) Show that the setAut(G) of all automorphisims of a group G is a group under the operation of function composition.
    (ii) If R* is multiplicative group of non zero real numbers,then show that the determinant mapping A→det A from GL(2,**R**) to **R*** is a group homomorphism with Kernal SL(2,**R**).

(OR)

(b) (i) If H is a subgroup of G and a, b belong to G then Show that $aH = bH$ if and only if $a^{-1}b \in H$

(ii) Show that the intersection of two normal subgroups of a group G is a normal subgroup of G.

11.(a) (i) Show that every finite integral domain is a field.

(ii) Show that the ring $Z[\sqrt{2}] = \{a + b\sqrt{2} / a, b \in Z\}$ is an integral domain.

(OR)

(b) If $\mathbf{R}[x]$ denote the ring of polynomials with real coefficients and $< x^2 + 1 >$ denote the principal ideal generated by $x^2 + 1$,then show that $\frac{R[x]}{< x^2 + 1 >}$ is a field.

12.(a) Let ϕ be a ring homomorphism from R to S then show that the mapping from R/Kerϕ to ϕ(R) given by r+ Ker$\phi \rightarrow \phi(R)$ is an isomorphism.

(OR)

(b)(i)If R is a ring with unity 1,then show that the mapping $\phi : Z \rightarrow R$ given by $n \rightarrow n.1$ is a ring homomorphism.

(ii) If n is an integer with decimal representation $a_k a_{k-1} \dots a_1 a_0$.Prove that n is divisible by 11 if and only if $a_0 - a_1 + a_2 - \cdots (-1)^k a_k$ is divisible by 11.

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