Code No.: 4782

FACULTY OF SCIENCE

M.Sc. IV Semester Examination, May/June 2010

MATHEMATICS

Paper I

(Advanced Complex Analysis)

Time : 3 Hours] [Max. Marks : 80

Answer **all** *questions.*

Section A – (Marks : 8 × 4 = 32)

1. Find the poles and residues of $\dfrac{1}{z^m(1-z)^n}$ where m,n positive integers.
2. How many zeros of the equation $z^4 + 8z^3 + 3z^2 + 8z + 3 = 0$ lie in the right half plane.
3. State and prove mean value property of a harmonic function.
4. State and prove Hadamard's three circle theorem.
5. Prove that $\prod_{n=1}^{\infty}\left(1+\dfrac{z}{n}\right).e^{-\frac{z}{n}}$ converges uniformly and absolutely on every compact set.
6. Prove that $\Gamma(z + 1) = z\,\Gamma(z)$.
7. Obtain Jensen's formula.
8. Show that the J-function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at $s = 1$ with the residue 1.

Section B – (Marks : 4 × 12 = 48)

9. (a) (i) State and prove Rouches theorem.

(ii) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{a+b\cos\theta}$, $a > b > 0$

Or

(b) (i) Show that $\int_0^{\infty} \dfrac{\sin x}{x}.dx = \dfrac{\pi}{2}$ by residue theory.

(ii) Explain the method to evaluate the integrals of the form $\int_0^{\infty} x^{\alpha} R(x)dx$ where $0 < \alpha < 1$.

[P.T.O.

10. (a) (i) State and prove Schwarz's theorem.

(ii) If u_1 and u_2 are harmonic in a region Ω then prove that $\int_{\upsilon} u_1^* du_2 - u_2^* du_1 = 0$ for every cycle υ which is homologous to zero in Ω.

Or

(b) (i) Suppose that $u(z)$ is harmonic for $|z| < R$, continuous for $|z| \le R$. Then prove that

$$u(a) = \frac{1}{2\pi} \int_{|z|=R} \frac{R^2 - |a|^2}{|z-a|^2} u(z)\, d\theta \text{ for all } |a| < R.$$

(ii) Show that every function f which is analytic in a symmetric region Ω can be written in the form $f_1 + rf_2$ where f_1, f_2 are analytic in Ω and real on the real axis.

11. (a) (i) Use Mittag-Leffeler's theorem to show that

$$\frac{\pi^2}{\sin^2 \pi z} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$$

(ii) State the prove Hurwitz theorem.

Or

(b) (i) Obtain the gamma function $\Gamma(z)$ from the canonical product

$$G(z) = \prod_{n=1}^{\infty} \left(1 + \frac{z}{n}\right) . e^{-z/n}.$$

(ii) Prove that $\Gamma(n+1) = n!$

12. (a) State and prove Hadmard's theorem.

Or

(b) (i) Prove that $\sigma = \text{Re}\, s > 1$

$$\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty} (1 - p_n^{-s})$$

(ii) Obtain the functional equation of the Riemann's zeta function.

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Code No. : 999

FACULTY OF SCIENCE
M.Sc. IV Semester Examination, May 2011
MATHEMATICS
Paper – I (401) Advanced Complex Analysis

Time: 3 Hours] [Max. Marks: 80

Instruction : *Answer all questions.*

SECTION – A **(8×4=32 Marks)**

1. How many roots does the equation
$z^7 - 2z^5 + 6z^3 - z + 1 = 0$
have in the disk $|z| < |$?

2. Find the poles and residues of the function $\frac{1}{(z^2-1)^2}$.

3. Suppose u is harmonic in a region Ω and $f(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$. Then show that :

   i) $f(z)dz = \left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy\right) + i\left(-\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy\right)$

   ii) For all cycles y which are homologous to zero in Ω, $\int_y \left(-\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy\right) = 0$

4. State and prove Hadmard's three circle theorem.

5. Show that

$$\frac{\pi^2}{\operatorname{Sin}^2(\pi z)} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$$

6. Show that :

   i) If g(z) an entire function then $f(z) = e^{g(z)}$ is entire and never zero.

   ii) If f(z) is any entire function which is never zero is of the form $f(z) = e^{g(z)}$, where g(z) is an entire function.

7. Show that the ζ – function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at S = 1 with residue 1 .

(This paper contains 2 pages)

P.T.O.

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Code No. : 999

8. Show that the function

$\xi(s) = \frac{1}{2} s(1-s)\pi^{-s/2}\,\Gamma(s/2)\,\zeta(s)$ is entire and satisfies $\xi(s) = \xi(1-s)$.

SECTION – B

(4×12=48 Marks)

9. a) i) State and prove the argument principle.

ii) If $a > 1$ prove that $\int_0^{\pi} \frac{d\theta}{a+\cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$

OR

b) i) State and prove Rouche's theorem.

ii) Show that

$$\int_0^{\infty} \frac{x^2\,dx}{x^4+5x^2+6} = \frac{\pi}{2}\left(\sqrt{3}-\sqrt{2}\right)$$

10. a) i) Show that the arithmetic mean of a harmonic function over concentric circles $|z| = r$ is a linear function of $\log r$, that is $\frac{1}{2\pi}\int_{|z|=r} u\,.\,d\theta = \alpha \log r + \beta$.

ii) Show that a non-constant harmonic function has neither maximum nor a minimum in its region of definition.

OR

b) i) Suppose $u(z)$ is harmonic for $|z| < R$ and continuous for $|z| \le R$. Then show that for all a with $|a| < R$, we have

$$u(a) = \frac{1}{2\pi}\int_{|z|=R} \frac{R^2-|a|^2}{|z-a|^2}\,u(z)\,d\theta$$

ii) If $f(z)$ is analytic in $|z| \le |$ and satisfies $|f| = 1$ on $|z| = 1$, show that $f(z)$ is rational.

11. a) i) Suppose that $f_n(z)$ is analytic in a region Ω_n for $n \ge 1$ and that the sequence $\{f_n(z)\}$ converges to a function $f(z)$ in a region. Ω, uniformly on every compact subset of Ω. Then show that

1) $f(z)$ is analytic in Ω

2) $f'_n(z)$ converges uniformly to $f'(z)$ on every compact subset of Ω.

Code No. : **9488**

FACULTY OF SCIENCE
M.Sc. IV Semester Examination, May/June 2012
MATHEMATICS
Paper – I : Advanced Complex Analysis

Time: 3 Hours] [Max. Marks:80

***Note**: Answer **all** questions.*

SECTION – A (8×4=32 Marks)

1. How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z|<1$?

2. Find the poles and residues of the function $\dfrac{1}{z^2 + 5z + 6}$.

3. If u and v harmonic in a region Ω then show that

   i) u+v is harmonic in Ω and

   ii) $f(z) = \dfrac{\partial u}{\partial x} - i\dfrac{\partial u}{\partial y}$ is analytic in Ω.

4. Show that if u (z) is harmonic for $|z| < R$, continuous for $|z| \le R$ then any analytic function F(z) having u(z) as its real part is given by

   $f(z) = \dfrac{1}{2\pi i}\displaystyle\int_{|\zeta|=R} \dfrac{\zeta + z}{\zeta - z}\, u(\zeta).\dfrac{d\zeta}{\zeta} + ic$, where c is a constant.

5. Show that the series $\zeta(z) = \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^z}$ converges for all z with Re (z) > 1.

6. Show that

   i) If g(z) an entire function then $f(z) = e^{g(z)}$ is entire and never zero.

   ii) If f (z) is any entire function which is never zero is of the form $f(z) = e^{g(z)}$, where g(z) is an entire function.

7. Show that an entire function of fractional order assumes every finite value infinitely many times.

8. For $\sigma = \mathrm{Re}\, s > 1$, show that $\dfrac{1}{\zeta(s)} = \displaystyle\prod_{n=1}^{\infty}(1 - p_n^{-s})$

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SECTION – B

(4×12=48 Marks)

9\. a) i) Show that

1) If f(z) has a simple pole at z= a then $\text{Res}_{z=a} f(z) = \lim_{z\to a} (z-a)\, f(z)$.

2) If f (z) has a pole of order h > 1 then $\text{Res}_{z=a} f(z) = \dfrac{\phi^{(h-1)}(a)}{(h-1)!}$.

Where $\phi(z) = (z-a)^h f(z)$ and $\phi^k(z)$ is its K^{th} derivative.

ii) Evaluate $\int_0^{\pi/2} \dfrac{dx}{a+\sin^2 x}, |\alpha|>1$, by residue theory.

OR

b) i) State and prove the argument principle

ii) Evaluate $\int_0^{\infty} \dfrac{x^{\frac{1}{3}}}{1+x^2}\, dx$.

10\. a) i) Show that the arithmetic mean of a harmonic function over concentric circles $|z| = r$ is a linear function of log r, that is,

$$\frac{1}{2\pi} \int_{|z|=r} u.\, d\theta = \alpha \log r + \beta$$

ii) Suppose u (z) is harmonic for $|z| < R$ and continuous for $|z| \le R$. Then show that for all α with $|a| < R$,

$$u(a) = \frac{1}{2\pi} \int_{|z|=R} \frac{R^2 - |\alpha|^2}{|z-\alpha|^2} u(z)\, d\theta$$

OR

b) i) Suppose U (θ) is piece wise continuous on [0, 2π] and Pu (z) its poisson integral. Then show that

1) P_U (z) is harmonic in $|z|<|$

2) $\lim_{z\to e^{i\theta_0}} P_U(z) = U(\theta_0)$, if $U(\theta)$ is continuous at θ_0.

ii) If f (z) is analytic in the whole plane and real on the real axis, purely imaginary on the imaginary axis, show that f(z) is odd.

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Code No. : 9488

11. a) i) State and prove Weierstrass' theorem.

ii) State and prove Mittag-Leffler's theorm.

OR

b) i) Show that the infinite product $\prod_{n=1}^{\infty}(1+a_n)$ converges absolutely if and only if the series $\prod_{n=1}^{\infty}\alpha_n$ is absolutely convergent.

ii) Show that $2^{2z-1}\,\Gamma_{(z)}\cdot\Gamma(z+\frac{1}{2})=\sqrt{\pi}\,(2z)$.

12. a) i) Obtain Jensen's formula.

ii) For the Riemann zeta function $\zeta(s)$, show that

$$\zeta(s)=2^{s}\,\pi^{s-1}\,\mathrm{Sin}\frac{\pi s}{2}\,\Gamma(1-s)\,\zeta(1-s).$$

OR

b) Show that the genus h and order λ of an entire function satisfy the double inequality $h\leq\lambda\leq h+1$.

Ayesha

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FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) IV - SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2015

ADVANCED COMPLEX ANALYSIS

PAPER - I

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Evaluate $\int_0^{\infty} \frac{\log(1+x^2)}{x^{1+\alpha}} dx$ $(0<\alpha<2)$. 5th M.

2. Suppose u is a Harmonic function in a regionΩ. Then (i) If v is a Harmonic conjugate of u in Ω then $*du = dv$ and (ii) For any cyclic α which is homologous to '0' in Ω, we have $\int_{\alpha}^{*} du = 0$.

3. Find the Taylor series of $f(z) = \frac{1}{1+z}$ at $z_0 = 1$. $\Rightarrow f(z) = \frac{1}{2} - \frac{(z-1)}{2^2} + \frac{(z-1)^2}{2^3}$

4. Show that the series $\sum \frac{1}{n^z}$ is convergent absolutely and uniformly in any bounded closed region in which $R(z) > 1$ and represent its derivative in series term.

5. Show that canonical product of $\sin \pi z = \pi z \prod_{1}^{\infty}\left(1 - \frac{z^2}{n^2}\right)$.

Section – B [illegible]

Answer the following questions in not more than **FOUR** pages each:

6. a) State and prove Residue theorem.

(OR)

b) If $P(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n$; $n \geq 1$ and $n \neq 0$ is a polynomial with complex coefficients then P(z) has 'n' zeros in the complex plane

7. a) i) Suppose $u = u(\theta)$ and $v = v(\theta)$ are piecewise continuous functions for $0 \leq \theta \leq 2\pi$ and $|z| < 1$, then (i) $P_{u+v} = P_u + P_v$ (ii) $P_{cu} = cP_u$ where c is constant (iii) $P_u \geq 0$ for $u \geq 0$ (iv) if $m \leq u \leq M$ then $m \leq P_u(z) \leq M$ and (v) $P_c = c$ where c is constant.

(OR)

b) Show that $\int_0^{\pi} \log \sin\theta \, d\theta = -\pi \log 2$.

8. a) If f(z) is analytic in the annular region $R_1 < |z-a| < R_2$. Then f(z) has a Laurent series expansion about 'a' that is:

$f(z) = \sum_{n=0}^{\infty} A_n (z-a)^n + \sum_{n=1}^{\infty} B_n (z-a)^{-n}$ where

$A_n = \frac{1}{2\pi i} \int_{|\xi - a| = r_2} \frac{f(\xi)}{(\xi - a)^{n+1}} d\xi$ $(n \geq 0)$ and

$B_n = \frac{1}{2\pi i} \int_{|\xi - a| = r_1}$ [illegible] for $n \geq 1$ and [illegible]

(OR)

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b) The infinite product $\prod_{n=1}^{\infty} (1+a_n)$, $1+a_n \neq 0\ \forall n$ and the series $\sum_{n=1}^{\infty} \log(1+a_n)$ converge simultaneously.

9. a) Suppose f(z) is analytic in the closed disk $|z| \leq \rho$ with $a_1, a_2, \ldots a_n$ as zeros in the disk and $f(0) \neq 0$ (i.e. $a_j \neq 0\ \ \forall J = 1,2,\ldots m$), then $\log|f(0)| = -\sum_{j=1}^{m} \log\left(\frac{\rho}{a_j}\right) + \frac{1}{2\pi}\int_0^{2\pi} \log|f(\rho e^{i\theta})|\,d\theta.$ Jensen's formula Theorem

(OR)

b) The ξ-function can be extended to a mermorphic function in the whole plane, whose only pole is a simple pole at S=1 with the residue 1.

10. (a) i) Show that $e^z = a.z^n$ has n roots for any $a > e$.

ii) How many roots of the equation $z^4 - 6z + 3 = 0$ have their modules between 1 and 2? Justify your answer.

(OR)

b) Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{a+\cos\theta}$ $(a > 1)$.

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Code No. 23/25/MS/4.1/ACA

FACULTY OF SCIENCE
M.Sc. (Mathematics) IV – SEMESTER
REGULAR/BACKLOG EXAMINATIONS, MAY 2014
ADVANCED COMPLEX ANALYSIS

PAPER – I

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer the following questions in not more than **ONE** page each: (5x4=20)

1. Define residue. If z = a is a pole of order m for f(z), then find residue of f(z) at z>a.
2. Suppose u is harmonic in a region Ω and v is a cycle homologous to zero in Ω, then $\int_r * du = 0$, where *du is the conjugate differential of du.
3. Show that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right) = {}^1/_2$.
4. Prove that ζ –function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at S = 1 with residue 1.
5. Prove that $\frac{\pi}{\sin \pi z} = \Gamma z \, \Gamma 1\text{-}z$

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) i) State and prove argument principle.
   ii) Evaluate the integral $\int_r \frac{f'(z)}{f(z)} dz$, when $f(z) = \frac{(z^2+1)^2}{(z^2+3z+2)^3}$ and is the circle |z|=3, taken in positive sense.

   (OR)

   b) i) Evaluate $\int_0^{\infty} \frac{1}{x^4+1} dx$.
   ii) Find the poles and residues of $(z) = \frac{1}{z^m(1-z)^n}$, m, n are positive integers.

7. a) State and prove Schwarz theorem.

   (OR)

   b) i) If f(z) is analytic in |z| ≤ 1 and satisfies |f| = 1 on |z| = 1, show that f(z) is rational.
   ii) If u is a harmonic in a region Ω and *du is the conjugate differential of du then for any cycle γ homologous to zero in Ω prove that $\int_\gamma * du = 0$.

8. a) State and prove Laurent's theorem.

   (OR)

   b) Prove that $\frac{x^2}{sin^2 \pi z} = \sum_{h=-\infty}^{\infty} \frac{1}{(z-h)^2}$.

9. a) State and prove Jensen's formula.

   (OR)

   b) For σ > 1, Res > 1, prove that $\frac{1}{\xi(s)} = \prod_{n=1}^{\infty}(1 - P_n^{-s})$.

10. a) State and prove Hadamard's theorem.

    (OR)

    b) i) State and prove residue theorem.
    ii) Evalua $\int_{|z|=2} \frac{e^{2z}}{(z-1)^n} dz$.

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Code No. Code No. 1841

FACULTY OF SCIENCE

M.Sc. (Mathematics) IV – SEMESTER REGULAR/BACKLOG EXAMINATIONS, APRIL 2015

ADVANCED COMPLEX ANALYSIS

PAPER – I

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer the following questions in not more than **ONE** page each: (5x4=20)

1. Define Pole. Find the pole and residue of the function $f(z) = \frac{Sin\ z}{z^4}$.
2. If u is harmonic in a Ω then prove that $f(z) = \frac{du}{dx} - \frac{idu}{dy}$ is analytic.
3. Evaluate the integral $\int_c \frac{1}{z(e^z-1)}\, dz$, where c:|z|=1.
4. Define entire function.
5. Write the entire function sin z as canonical product.

sin z = 0

sin z = nπ

z = nπ

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) i) State and prove residue theorem.

ii) If z = a in a pole of order m for f(z), then prove that

$$\underset{z=a}{Residue\ at}\ f(z) = \frac{1}{(n+1)!} \lim_{z\to a} \frac{d^{m-1}}{dz^{m-1}} [(z-a)^m f(z)]$$

(OR)

b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{Sin\ mz}{z}\, dz,\quad m > 0.$ (3) 2π or π/2

7. a) State and prove Poisson's formula.

(OR)

b) State and prove Schwartz theorem.

8. a) State and prove Mittag – Lefler's theorem

(OR)

b) Prove that a necessary and sufficient condition for the absolute convergence of the product $\prod_{n=1}^{\infty}(1 + a_n)$ is the convergence of the series $\sum_{n=1}^{\infty} |a_n|$.

9. a) Define Reimann – zeta function. Prove that for σ = Res (s)>1, then $\frac{1}{\zeta(S)} = \prod_{n=1}^{\infty}(1 - P_n^{-s})$.

(OR)

b) Prove that the genus and order of an entire function satisfy the double inequality $n \leqq \lambda \leq n + 1$.

10. a) Using the Mittag – Lefler's theorem prove that $\frac{\pi^2}{Sin^2 \pi z} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$

(OR)

b) State and prove the functional equation of the Reimann – zeta function.

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Code No. Code No. 1841

FACULTY OF SCIENCE
M.Sc. (Mathematics) IV – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017
ADVANCED COMPLEX ANALYSIS
PAPER – I

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer the following questions in not more than **ONE** page each: (5x4=20)

1. Find the pole and residue of $f(z) = \frac{e^z}{(z-a)(z-b)}$.
2. Suppose u(z) is a harmonic function in a region Ω, Then (i) $f(z)\,dz = du + idu^*$ where $^*du = \frac{-\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ and du is the total derivative of u. Then $du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$. And (ii) $f(z) = \frac{\partial u}{\partial x} - \frac{i\partial u}{\partial y}$ is analytic in the region Ω.
3. If$(z) = \frac{1}{(z+1)(z+3)}$, expand $f(z)$ in a laurent series valid for 1<l z l<3.
4. An entire function of fractional order assumes every finite value unlimitedly many times.
5. Let$(z) = \frac{1}{[q(z)]^2}$, where q(z) has a sample zero at zo. Then zo is a pole of order 2 of f(z) and the reduce at this pole is given by $b_1 = \frac{-q^{11}(zo)}{[q^1(zo)]^3}$

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) Evaluation of integral theorem$\int_{\infty}^{0} R(x)ds$.

(OR)

b) Prove that $\int_0^{2\pi} \frac{sin^2\theta}{a+b\,cos\theta}d\theta = \frac{2\pi}{b^2}\left(a - \sqrt{a^2-b^2}\right)$

7. a) State and prove Schwartz theorem on Poisson.

(OR)

b) Let f(z) be an analytic function in a domain Ω containing a segment of the x-axis and is symmetric to that axis, then $\overline{f(z)} = f(z), z \in \Omega$ if and only if f(x) is real for even point on the segment of x-axis.

8. a) If f(z) is analytic on open disc $\Delta = \{z/|z - z0| < p\} \leq \Omega$ then the series $\sum_{n=\partial}^{\infty} \frac{f^{(n)}(z0)}{n!}(z - z0)^n$ answer to f(z) ∀ z∈ Δ

(OR)

b) State and prove Lauren's theorem.

9. a) Suppose f(z) is analytic in the closed disc $|z| \leq \rho$ when zero's at $a_1, a_2, \ldots.. a_m$ in the disc $|z| \leq \rho$ and f(z)=0.

Then $\log|f(z)| = -\sum_{j=1}^{m} log\left|\frac{\rho^2 - \bar{a}_j z}{\rho(z-a_j)}\right| + \frac{1}{2\pi}\int_0^{2\pi} \rho_e\left[\frac{\rho e^{i\theta}+z}{\rho e^{i\theta}-z}\right] log|f(\rho e^{i\theta})|d\theta$

P.T.O

(OR)

b) If $s = \sigma + it$ aim $\sigma > 1$ or $Re(s) > 1$, then $E(s) = \prod_{n=1}^{\infty}\left(1 - \frac{1}{p_n^s}\right)^{-1}$

10.a) Evaluate two integral $\int_0^{2\pi} \frac{d\theta}{2-sin\theta}$

(OR)

b) Prove that $\int_0^{\pi} \frac{d\theta}{(a+cos\theta)^2} = \frac{\pi a}{(a^2-1)^{3/2}}$ $(a > 1)$.

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032-07-52-001

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Code No.: 5643

FACULTY OF SCIENCE

M.Sc. IV Semester Examination, April/May 2009

MATHEMATICS

Paper II

(General Measure Theory)

Special Note : In case of Old Batch students, marks scored would be converted to maximum marks of 100

Time : 3 Hours]

[Max. Marks : 80

Answer **all** *questions in* Section A *and* Section B.
Each question carries 4 *marks in* Section A *and* 12 *marks in* Section B.

Section A – (Marks : 8 × 4 = 32)

1. Suppose $(x_1 \beta_1 \mu)$ is a measure space and $\{E_i\}$ is a sequence in β. Then prove that
$$\mu\left(\bigcup_{i=0}^{\infty} E_i\right) \subseteq \sum_{i=1}^{\infty} \mu(E_i).$$

2. State and prove Lebesgue dominated convergence theorem.

3. Define a signed measure on a measure space (α, β). Prove that every measurable subset of a positive set is positive.

4. If $\overset{+}{\gamma}$ and $\overset{-}{\gamma}$ are the positive and negative parts of a signed measure γ and E is a measurable set then prove that $-\overset{-}{\gamma}E \leq \gamma E \leq \overset{+}{\gamma}E$ and $|\gamma E| \leq |\gamma|(E)$.

5. Suppose A is an algebra of subsets of α and μ is a measure in A. Define the outer measure μ^x induced by μ. Suppose $E \subset X$ then prove that given any $\epsilon > 0$ there exists a set $A \in A_\sigma$ with $E \subset A$ and $\mu^x \subset A_1 \subset \mu^x \subset E_1 + E$.

6. Suppose EC × xy and $x \in X$. Define E_x the x cross section of E. Prove that
(a) $\psi_{E_x}(y) = \psi_E(x, y)$ (b) $(\overline{E})_x = (\overline{E_x})$

7. Suppose $A \in A$ then prove that $\mu(A) = \mu_x(A \cap E) + \mu^x(A \cap \overline{E}) \forall ECX$.

8. Let μ be a measure on a σ-algebra A of subsets of X and let m be a collection of such sets of X which is closed under countable unions prove that
$\beta = \{B : B = A \Delta M, A \in A, M \in m\}$ is the smallest -σ algebra contains both A and m.

P.T.O.

30 (10)

Section B – (Marks : 4 × 12 =48)

9. (a) Suppose (X, β) is a measurable space and $E \in \beta$. For a function $f : E \to [-\alpha, \alpha]$ prove that the following statements are equivalent.
   (i) $\{x \in E : f(x) > \alpha\} \in \beta$ for all $\alpha \in R$
   (ii) $\{x \in E : f(x) \geq \alpha\} \in \beta$ for all $\alpha \in R$
   (iii) $\{x \in E : f(x) < \alpha\} \in \beta$ for all $\alpha \in R$
   (iv) $\{x \in E : f(x) \leq \alpha\} \in \beta$ for all $\alpha \in R$.

Or

(b) Suppose (X, β) is a measurable space and f is a non-negative β. measurable function on X. Then prove that there exists an increasing sequence of non negative simple functions $\{\phi_n\}$ such that $\lim_{n \to \infty} \phi_n(x) = f(x)$ for all $x \in X$.

10. (a) (i) Suppose γ is a signed measure on a measurable space (X, β). If $E \in \beta$ is such that $0 < \gamma(E) < \infty$ then prove that E has a positive set A with $\gamma(A) > 0$.

(ii) State and prove Hahn-decomposition theorem.

Or

(b) State and prove Radon-Nikodym theorem.

11. (a) Suppose ζ is a semi algebra of subsets of X and A is the algebra generated by ζ. Let $\mu_0 : \zeta \to [0, \alpha]$ be a non-negative real valued set function satisfying the conditions
   (i) $\mu(\phi) = 0$
   (ii) If $C \in \zeta$ and $C = \bigcup_{j=1}^{n} C_j$; where C_j's are pairwise discount then

$$\mu_0(C) = \sum_{j=1}^{n} \mu_0(C_j)$$

   (iii) If $C \in \zeta$ and $C = \bigcup_{j=1}^{\infty} C_j$ where $\{C_j\}$ is a pairwise discount sequence in ζ then

$\mu_0(C) < \sum_{i=1}^{\infty} \mu_0(C_j)$. Then show that there exists a measure ζ on A such that $\zeta(C) = \mu_0(C) \forall C \in \zeta$.

Or

(b) State and prove Fubini's theorem.

12\. ~~(a)~~ (i) Suppose E and F are disjoint sets then prove that

$$\mu_\infty(E) + \mu_\infty(F) \le \mu_\infty(EUF) \le \mu_\infty(E) + \mu_\infty(F) \le \mu^\infty(EUF) \le \mu^n(E) + \mu^\infty(F)$$

(ii) If $\{E_i\}$ is sequence of pairwise disjoint sets then prove that

$$\sum_{i=1}^{\infty} \mu_\infty(E_i) \le \mu_\infty\left(\bigcup_{i=1}^{\infty} E_i\right)$$

Or

(b) State and prove Coratheodory theorem.

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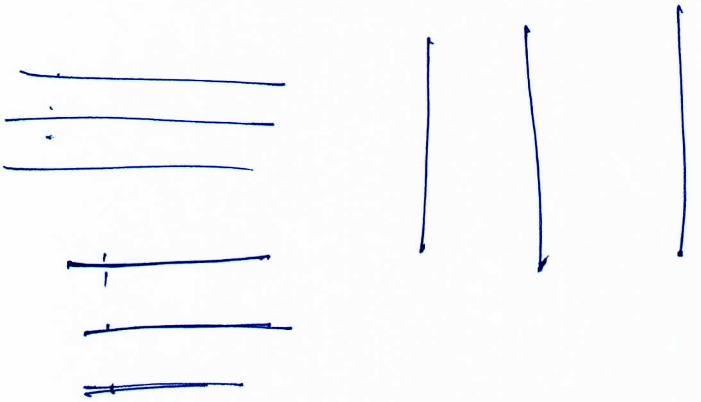

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FACULTY OF SCIENCE

M.Sc. IV Semester Examination, May/June 2010

7

MATHEMATICS

Paper II

(General Measure Theory)

Time : 3 Hours] [Max. Marks : 80

*Answer **all** questions in section A and section B.*
Each question carries 5 marks in section A and 15 marks in section B.

Section A - (Marks : 8×4 =32)

1. Suppose (x, β, μ) is a measure span and $\{E_i\}$ is a sequence in β such that $E_i \supset E_{i+1} \forall i = 42$. Prove that

$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{i \to \infty} \mu(E_i).$$

2. State and prove Lebesque dominated convergence theorem.
3. Define postive, negative and null set with respect to a signed measure υ on a measurable span distinguish between a null set and a set of measure zero.
4. State and prove Hahn decomposition theorem.
5. Suppose A is on algebra of subsets of x and μ is a measure on A. Define the outer measure μ^* induced by μ. Also prove that if $A \in A$ then $\mu^*(A) = \mu(A)$.
6. Define a semi algebra of subsets of x. Prove that the collection R of measurable rectangles is a semi-algebra.
7. Let $A, B \in A$ with $\mu^*(A - E) < \infty$ and $\mu^*(B - E) < \infty$. Then prove that
   (i) $A \subset B \Rightarrow \mu(A) - \mu^*(A - E) \leq \mu(B) - \mu^*(B - E)$
   (ii) $E \subset AC \Rightarrow \mu(A) - \mu^*(A - E) = \mu(B) - \mu^*(B - E)$
8. Let (x, s) be a metric space. If μ^* is a metric outer measure on subsets of x then prove that every closed set is a μ^*-measurable set.

Section B - (Marks : 4×12 =48)

9. (a) Suppose (v, β) is a measurable space and $E \in \beta$ and $\{f_n\}$ is a sequence of β measurable functions defined on β. Then prove that $\sup_{n \geq 1} f_n$, $\inf_{n \geq 1} f_n$, $\limsup_{n \to \infty} f_n$, $\liminf_{n \to \alpha} f_n$ are also β measurable also prove that if $\lim_{n \to \infty} f_n = f$ exists from f is also β measurable.

Or

(b) State and prove Fatou's lemma. Hence deduce monotone convergence theorem.

2) (b) Suppose Γ is a set of real valued fun. on X. Then define [P.T.O.
Carathedary outer measure w.r.t Γ. If μ^* is a Carathedory outer measure w.r.t ~~Γ th~~ ~~plz~~ ~~Sue~~ ~~y~~ ~~fun~~ ~~in~~ Γ is a
μ^* - measurable fun.

10. (a) (i) State and prove lebesgue decomposition theorem.

(ii) Suppose $1< P<\infty$ and $f\in L^p$ and $g \in L^p$. Then Prove that $\|f+g\|_p \leq \|f\|_p + \|f\|_p$.

m in

Or

(b) Suppose p and q are positive real numbers such that $\frac{1}{p}+\frac{1}{q}=1$

Let $g \in L^q(\mu)$ where (x, β, μ) is a measure space

Let $f_g : L^q(\mu) \to R$ be defined by $f_g(f) = \int_x fg d\mu$ for $f \in^{\prime} L(\mu)$

Then prove that

(i) f_g is a linear functional

(ii) f_g is bounded linear functional

(iii) $\|f_g\| = \|g\|_q$

11. (a) State and prove carathodory extension theorem.

Or

(b) Suppose $(x, \boldsymbol{A}, \mu$ and $(y \beta, \gamma)$ are complete measure spaces and R is the clean f all measurable rectangles of the product space $x \times y$ supose $E \in R_{\sigma\delta}$ with $(\mu \times \gamma)$ $LE < \infty$. Prove that the function $g : x \to [0, \infty]$ defined by

$g(x_1 = \gamma LE_x) \;\forall\; n \in x$ is a measurable function Mx Also prove that $\int_x g d\mu = (\mu \times \gamma) c \in 1$.

12. (a) (i) Suppose μ is a measure on an algebra $\boldsymbol{A}$ such that μ^* and μ_* are respectively the outer and inner measures induced by μ. Suppose β is the clen of all μ measurable. If $EC\, x$ is such that $\mu(E) < \infty$ then prove that there exists a $H \in \boldsymbol{A}_{\sigma\delta}$ such that $\bar{\mu}(H) = \mu_*(E)$ where $\bar{\mu}$ is the restriction of μ^* to β.

(ii) Suppose ECx with $\mu^*(E) < \infty$. Then prove that E is μ^*-measurable of and only of $\mu^* CE1 = \mu_* CE1$.

Or

(b) Let μ be a measure on a σ-algebra $\boldsymbol{A}$ of subsets of X and let m be a collection of subsets of X which is closed under countable unions and which has the property that for each $A \in \boldsymbol{A}$ with $ACM \in m$ on we have $\mu(A) = 0$. Then prove that there is an extension $\bar{\mu}$ of μ to the smallest σ-algebra β containing $\boldsymbol{A}$ and m such that $\mu(M) = 0$ for each $M \in m$.

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FACULTY OF SCIENCE

M.Sc. IV Semester Examination, May 2011

MATHEMATICS

Paper – II (402) General Measure Theory

Time: 3 Hours]

[Max. Marks : 80

*Note : Answer **all** questions.*

SECTION – A **(8×4=32 Marks)**

1. Suppose $(X,\mathcal{B},\mu)$ is a measure space. If $E_i \in \mathcal{B}, \mu(E_1) < \infty$ and $E_i \supset E_{i+1}$ then prove that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu(E_n)$.

2. Let f be a non-negative measurable function. Then prove that there is a sequence $\langle \varphi_n \rangle$ of simple functions with $\varphi_{n+1} \ge \varphi_n$ such that $f = \lim \varphi_n$ at each point of X.

3. Let $(X, \mathcal{B}, \mu)$ be a finite measure space and g an integrable function such that for some constant $M > 0$, $\left| \int g\varphi \, d\mu \right| \le M \|\varphi\|_p$ for all simple functions φ. Then prove that $g \in L^q[\mu]$.

4. State and prove Lebesgue's decomposition theorem.

5. Prove that the collection of all measurable rectangles is a semialgebra.

6. Let μ be a measure on an algebra $\mathcal{A}$, μ^* the outer measure induced by μ and E any set. Then for any $\in > 0$ prove that there is a set $A \in \mathcal{A}_\sigma$ with $E \subseteq A$ and $\mu^*(A) \le \mu^*(E) + \in$.

7. Let B be a μ^* - measurable set with $\mu^*(B) < \infty$. Then prove that $\mu_*(B) = \mu^*(B)$.

8. If $A \in \mathcal{A}$ then prove that $\mu(A) = \mu_*(A \cap E) + \mu^*\left(A \cap \tilde{E}\right)$ for any set E.

(This paper contains 3 pages)

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SECTION – B (4×12=48 Marks)

9\. a) i) State and prove Fatou's lemma.

ii) State and prove Monotone convergence theorem.

OR

b) Suppose f and g are integrable functions and E is a measurable set. Then prove the following statements.

i) $\int_E (c_1 f + c_2 g) = c_1 \int_E f + c_2 \int_E g$

ii) If $|h| \leq |f|$ and h is measurable then h is integrable.

iii) If $f \geq g$ a.e then $\int f \geq \int g$.

10\. a) i) State and prove Hahn's decomposition theorem.

ii) Give an example to show that Hahn's decomposition need not be unique.

OR

b) State and prove Radon-Nikodym theorem.

11\. a) Prove that the $\mathcal{B}$ of μ^* measurable sets is a σ algebra. If μ^* restricted to $\mathcal{B}$ is denoted by $\bar{\mu}$ then prove that $\bar{\mu}$ is a complete measure on $\mathcal{B}$.

OR

b) i) Let E be a set for which $\mu \times V(E) = 0$. Then prove that for almost all x, $V(E_x) = 0$

ii) Let E be a measurable subset of X × Y such that $(\mu \times V)(E) < \infty$. Then prove that E_x is a measurable subset of Y for almost all x, g defined by $g(x) = V(E_x)$ is a measurable function defined for almost all x and $\int g d\mu = (\mu \times V)(E)$.

12) a) let μ be a measure on a σ-algebra $\mathcal{A}$ of subset of X & $\mathcal{M}$ be a col of subsets of X, which is closed under countable union & which has the property that for each $A \in \mathcal{A}$ with $A \subseteq M$, $M \in \mathcal{M}$ we've $\mu(A) = 0$. Then P/T there is an ext^n $\bar{\mu}$ of μ to the smallest σ-algebra $\mathcal{B}$ containing $\mathcal{A}$ & $\mathcal{M}$ ∋ $\bar{\mu}(M) = 0$ for each $M \in \mathcal{M}$.

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FACULTY OF SCIENCE
M.Sc. IV Semester Examination, May/June 2012
MATHEMATICS
(Paper – II) General Measure Theory

Time: 3 Hours] [Max. Marks : 80

***Note**: Answer **all** questions.*

SECTION – A (8×4=32 Marks)

1. If $E_i \in \mathcal{B}$ then prove that $\mu\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} \mu E_i$.

2. If μ is a complete measure and f is measurable function and if $f = g$ a. e, then prove that g is measurable.

3. Define positive, negative and null set with respect to a signed measure ν on a measurable span distinguish between null set and a set of measure zero.

4. Let E be a measurable set such that $0 < \nu E < \infty$. Then prove that there is a positive set A contained in E with $\nu A > 0$.

5. If $<E_i>$ is a sequence of measurable sets and if $E = \cup E_i$, then for any set A prove that $\mu^*(A \cap E) = \Sigma \mu^*(A \cap E_i)$.

6. If $A \in \mathcal{A}$ and if $< A_i >$ is any sequence of sets in $\mathcal{A}$ such that $A \subset \bigcup_{i=1}^{\infty} A_i$, then prove that $\mu A \leq \sum_{i=1}^{\infty} \mu A_i$.

7. Define inner measure μ_* and if $E \in \mathcal{A}$ then prove that $\mu_* E = \mu E$.

8. If $A \in \mathcal{A}$, then prove that $\mu A = \mu_*(A \cap E) + \mu^*(A \cap \tilde{E})$.

(This paper contains 2 pages)

P.T.O.

SECTION – B (4×12=48 Marks)

9. a) State and prove Facton's Lemma.

OR

b) State and prove Lebesgue Convergence theorem.

10. a) State and prove Radon – Nikodym theorem.

OR

b) If $(X, \mathcal{B}, \mu)$ be a finite meaure space and g is an integrable function such that for some constant M, $\left|\int g\phi\, d\mu\right| \le M\|\phi\|_p$ for all simple functions ϕ, then prove that $g \in L^q$.

11. a) Let μ be a σ-finite measure on an Algebra $\mathcal{A}$ and let μ^* be the outer measure generated by μ. Then prove that a set E is μ^* measurable if and only if E is a proper difference $A \sim B$ of a set A in $\mathcal{A}_{\sigma\delta}$ and a set B with $\mu * B = 0$. Each set B with $\mu * B = 0$ is contained in a set C in $\mathcal{A}_{\sigma\delta}$ with $\mu * C = 0$.

OR

b) State and prove Fubini's theorem.

12. a) Let E and F be disjoint sets. Then prove that

$$\mu_* E + \mu_* F \le \mu_*(E \cup F) \le \mu_* E + \mu^* F \le \mu^*(E \cup F) \le \mu^* E + \mu^* F$$

OR

b) If μ^* is a caratheodory outer measure with respect to Γ, then prove that every function in Γ is μ^* – measurable.

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FACULTY OF SCIENCE

M.Sc (MATHEMATICS) IV – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2013

GENERAL MEASURE THEORY

PAPER – II

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Suppose (x, β, μ) is a measure space if $E_i \in \beta$ then prove that $\mu(\cup_{i=0}^{\infty} E_i) \leq \sum_{i=1}^{\infty} \mu(E_i)$.
2. Determine Positive, Negative and Null set with respect to a signed measure ϑ on a measurable span distinguish between a null set and a set $\mathcal{A}$ measure zero.
3. Define semi-algebra of subsets of X. Prove that the collection R of measurable rectangles is a semi-algebra.
4. If $A \in \mathcal{A}$, then prove that $\mu(A) = \mu^*(A \cap E) + \mu^*(A \cap \tilde{E})$.
5. [illegible] $g \in L^q(\mu)$. Let F be the linear functional on $L^p(\mu)$ defined by $F(f) = \int fg\,d\mu$ [illegible] that $\|F\| = \|g\|_q$.

Section – B

Answer the following questions in not more than **FOUR** pages each.

① Sum

② min

③ Hol

④ E∪(E) m'ble

⑤ max/min

6. a) State and prove Fatou's Lemma.

(OR)

b) Suppose (x, β) is a measurable space and f be an extended real valued function defined on X then prove that the following statements are equivalent.

i) $\{x/f(x) > \alpha\} \in \beta$ for all $\alpha \in R$

ii) $\{x/f(x) \geq \alpha\} \in \beta$ for all $\alpha \in R$

iii) $\{x/f(x) < \alpha\} \in \beta$ for all $\alpha \in R$

iv) $\{x/f(x) \leq \alpha\} \in \beta$ for all $\alpha \in R$

7. a) State and prove Radon-Nikodym theorem.

(OR)

b) State and prove Hahn-decomposition theorem.

8. a) Let μ be a measure on an algebra $\mathcal{A}$, μ^* be the outer measure induced by μ and E is any set. Then prove that for $\in$ [illegible] there is a set $A \in \mathcal{A}$ with $E \subset A$ and $\mu^*(A) \leq \mu^*(E) + \in$. Further show that there is also a set $B \in \mathcal{A}_{\sigma\delta}$ with $E \subset B$ and $\mu^*(E) = \mu^*(B)$.

(OR)

b) [illegible]

7\. a) Let E and F be two disjoint sets then prove that $\mu_*(E) + \mu_*(F) \leq \mu_*(E \cup F) \leq \mu^*(E) + \mu^*(F) \leq \mu^*(E \cup F) \leq \mu^*(E) + \mu^*(F)$.

(OR)

b) If Γ is a set of real valued functions on X, then define a Caratheodory outer measure with respect to Γ. If μ^* is a Caratheodory outer measure with respect to Γ, then show that every function in Γ is a μ^* measurable function.

8\. a) Let (X, β) be a measurable space and $\{f_n\}$ is a sequence of measurable functions defined on β. Then prove that: i) $\sup_{n\geq1} f_n$ ii) $\inf_{n\geq1} f_n$ iii) $\lim\sup f_n$ iv) $\lim\inf_{n\to\infty} f_n$ are also measurable function.

(OR)

b) Suppose $E \in R_{\sigma\delta}$ with $(\mu \times \delta)(E) < \infty (finite)$ then [illegible] the function defined [illegible] $(E_x) \forall x \in X$ is a measurable function [illegible] $\int g d\mu = (\mu \times \gamma)(E)$.

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FACULTY OF SCIENCE
M.Sc. (Mathematics) IV – SEMESTER
REGULAR/BACKLOG EXAMINATIONS, MAY 2014
GENERAL MEASURE THEROY

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PAPER – II

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer the following questions in not more than **ONE** page each: $\mu(A) \leq \mu(B)$ (5x4=20)

1. Define a measure μ on a measurable space $(x, \mathcal{B})$, prove that measure μ is monotonic.
2. Suppose $(x, \mathcal{B}, \mu)$ is a measure space and f is an integrable function on, x with respect to μ. Suppose γ is define on $\mathcal{B}$ by $\gamma(E) = \int_E f\,du\ \forall E \in \mathcal{B}$. Prove that γ is a signed measure on $(x, \mathcal{B})$.
3. Define a μ^* measurable set. If E_1 and E_2 are μ^* measurable. Prove that $E_1 \cup E_2$ is also a μ^* measureable set. RA
4. Suppose μ^* and μ_* are respectively the outer and inner measure induced by a measure μ on an algebra $\mathcal{A}$ of sub sets of X. Prove that $\mu_*(E) \leq \mu^*(E)\ \forall E \subset X$.
5. State and prove Holder's inequality.

RA

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) Suppose $(x, \mathcal{B})$ is a measurable space and $\{f_n\}$ is a sequence of β measurable functions defined on $E \in \mathcal{B}$. Prove that:
i) max $(f_1, f_2, \ldots f_n)$, ii) Min$(f_1, f_2, \ldots, f_n)$ iii) $\sup_{n \geq 1} f_n$ iv) $\inf_{n \geq 1} f_n$, v) $\limsup_{n \to \infty} f_n$, vi) $\liminf_{n \to \infty} f_n$ are all $\mathcal{B}$-measurable.

(OR)

b) i) Suppose $(x, \mathcal{B}, \mu)$ is a measure space and f, g are non-negative measurable functions defined on $E \in \mathcal{B}$. If a, b are non-negative constants prove that: $\int_E (af + bf)\,d\mu = a\int_E f\,d\mu + b\int_E g\ldots$
ii) State and prove Lebesgue convergence theorem.

7. a) State and prove Jordon-Decomposition theorem.

(OR)

b) Prove that a normed linear space X is complete if and only if every absolutely summable series in X is summable in X. RA

8. a) Suppose μ^* is an outer measure on pex, and $\mathcal{B}$ is the class of all μ^*-measurable sets. If $\bar{\mu}$ is a complete measure on $\mathcal{B}$. RA

(OR)

b) i) Suppose $(x, \mathcal{A}, \mu)$ and (y, β, γ) are complete measure spaces and R is the collection of all measurable rectangles in $X * Y$. (2 Thm
ii) Suppose $\mu_0 : R \to [0, \infty]$ is a mapping defined by $\mu_0(A \times B) = \mu(A, \gamma(B))\ \forall\ A \times B \in R$, prove that $\mu_0(\emptyset) = 0$ and $\mu_0(A \times B) = \sum_{i=1}^{\infty} \mu_0(A_i \times B_i)$.

9. a) Suppose $A, B \in \mathcal{A}$ with $\mu^*(A - E) < \infty$, $\mu^*(B - E) < \infty$, if $A \subset B$, prove that:
i) $\mu(A) - \mu * (A - E) \leq \mu(B) - \mu * (B - E)$
In addition if E⊂A⊂B prove that:
ii) $\mu(A) - \mu * (A - E) = \mu(B) - \mu * (B - E)$ 5
iii) $\mu_\infty(E) = \mu(A) - \mu^*(A - E)$

(OR)

b) State and prove Corathrodery outer measure theorem.

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Code No. 23/25/MS/4.2/GMT

10.a) i) Suppose (x,$\mathcal{A}$,μ) and (y, $\mathcal{B}$,γ) are complete measure spaces and R is the dim of all measurable rectangles in R. If $E \in R_{\sigma\delta}$. Prove that E is a measurable subset of y for any $x \in X$.

ii) If $E \subset X \times y$ with (μxγ)(E) = 0. Prove that γ(E_x) = 0 for all most all $x \in X$.

III

(OR)

b) Suppose (x, $\mathcal{B}$) is a measurable space and {μ_n} is a sequence of measures that converge set-wise to a measure μ on $\mathcal{B}$. Let{f_n} be a sequence of non-negative measurable functions that converge point wise to a limit of function f. Prove that $\int_X f d\mu \leq \lim\inf \int_X f_n d\mu_n$. — I (17) Thm

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FACULTY OF SCIENCE

M.Sc. (Mathematics) IV – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017

GENERAL MEASURE THEORY

PAPER – II

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer the following questions in not more than **ONE** page each: (5x4=20)

1. State and prove Lebesque dominated convergence theorem.
2. If $f \in L^p(\mu), g \in L^q(\mu), \frac{1}{p} + \frac{1}{q} = 1, 1\angle p \angle \infty$ then prove that $\int |fg| d\mu \leq \| f \|_p \ \| g \|_q$.
3. If $\angle E_i >$ is a sequence of measurable sets and of $E = \cup E_i$, then for any set A. Prove that $\mu^*(A \cap E) = \sum \mu^*(A \cap E_i)$
4. Define inner measure μ_* and if $E \in \mathcal{A}$. Then prove that $\mu_*(E) = \mu(E)$.
5. Define the following concepts:
   i) Measure ii) Complete measure iii) δ- finite measure.

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) Suppose f and g are integral functions and E is a measurable set. Then prove the following statements.
   i) $\int_E (af + bg) = a\int_E f + b\int_E g$
   ii) If $|h| \leq |f|$ and h is measurable then h is integrable.
   iii) If $f \geq g$ a.e. then $\int_E f \geq \int_E g$.

   (OR)

   b) Suppose (x, β, μ) be a measurable space and f be a non-negative measurable function then prove that there exists an increasing sequence of non negative simple functions $\{\phi_n\}$ such that $\lim_{n\to\infty} \phi_n(x) = f(x)$ for all $x \in X$.
7. a) State and prove Riesz-Representation theorem.

   (OR)

   b) Let υ be a signed measure and E be a measurable set with $0 \angle \upsilon(E) \angle \infty$. Prove that there is a positive set A contained in E with $\upsilon(A) > 0$.
8. a) State and prove Carathodory Extension theorem.

   (OR)

   b) i) Let $\{(A_i \times B_i)\}$ be a countable disjoint collection of measurable rectangles whose union is a measurable rectangle A x B. Then prove that $\lambda(A \times B) = \sum \lambda(A_i \times B_i)\}$.
   ii) Let $x \leftarrow X$ and $E \in R_{\sigma\delta}$ then prove that E_x is a measurable subset of Y.
9. a) Let μ be a measure on an algebra $\mathcal{A}$. Then prove the following
   i) $\mu_*(E) \leq \mu^*(E)$ for all sets E
   ii) $\mu_*(E) = \mu(E)$ for $E \in \mathcal{A}$.
   iii) If $E \leq F$ then $\mu_*(E) \leq \mu_*(F)$.

   (OR)

   b) i) Let B be a μ^* measurable set with $\mu^*(B) < \infty$ then show that $\mu_*(B) = \mu^*(B)$.
   ii) Let $\{A_n\}$ be a disjoint sequence of sets in $\mathcal{A}$ then prove that $\mu_*(E \cap \cup_{n=1}^{\infty} An) = \sum_{n=1}^{\infty} \mu_*(E \cap An)$.
10. a) State and prove Lebseque –Decomposition theorem.

    (OR)

    b) Let μ be a σ-finite measure on an algebra $\mathcal{A}$ and let μ^* be the outer measure generated by μ. A set E is μ^*-measurable if and only if E in the proper difference A~B of the set A in $\mathcal{A}\sigma\delta$ and a set B with $\mu^*(B)=0$. Measure each set B with $\mu^*(B)=0$ is contained in a set c in $\mathcal{A}\sigma\delta$ with $\mu^*(C)=0$.

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FACULTY OF SCIENCE

M.Sc. IV Semester Examination, May/June 2010

MATHEMATICS / APPLIED MATHEMATICS

Paper V (A)

(Calculus of Variations (405))

Time : 3 Hours]

[Max. Marks : 80

Answer **all** *questions.*

Section A – (Marks : 8×4 =32)

1. Define a functional, a linear functional and a geodesic.

2. Distinguish between strong variation and weak variation with suitable example.

3. Find the extremals of the functional

$$u[y(x), z(x)] = \int_0^{\pi/2} \left(y^{12} + z^{12} = 2yz\right)dx$$

Subject to $y(0) = 0,\ \ y(\frac{\pi}{2}) = 1,\ \ z(0) = 0,\ \ z(\frac{\pi}{2}) = -1$

4. Find the extremum of the functional

$$V[y, x] = \int_0^{\pi/4} \left(y^{12} - y^2\right) dx \ ; \ y(0) = 0.\ y(\frac{\pi}{2}) = 1$$

5. State the isoperimetric problem and indicate its solution

6. Find the Euler Ostrogradsky equation for the functional

$$S[z(x,y)] = \int_D \int \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \ dx\, dy.$$

7. Use Harmiltons equations to find the equations of motion of a projectile in space.

8. Derive the differential equation of motion of simple pendulum using Lagrange's equation.

[P.T.O.

(d) 53

Section B – (Marks : 4×12 =48)

9\. (a) State the simplest variational problem. Solve the variational problem

$$\int_1^e \left(x y'^2 + y y' \right) dx; \; y(1)=0, \; y(e)=1$$

Or

(b) (i) State and prove the fundamental Lemma of calculus of variations.

(ii) Prove that the shortest distance between two points in a plane is a straight line.

10\. (a) Using the variational principle, find the curve connecting two given points A and B that is traversed by a particle sliding from A to B in the shortest time.

Or

(b) Find the curve passing through the points (x_1, y_1), (x_2, y_2) which when rotated about the X-axis gives a minimum surface area.

Or

11\. (a) Derive the Euler–Ostrogradsky equation for the function

$$v[z(x,y)] = \iint_D F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right) dx\, dy .$$

Or

(b) Show that the sphere is the solid figure of revolution, which for a given surface area, has maximum volume.

12\. (a) Derive the differential equation of the free vibrations of a string using the variational principle.

Or

(b) Derive the Euler–Poisson equation.

26 9 11 40 40

54 14

10. a) Find the extremum of the functional

$$V[y, x] = \int_0^{\pi/4} \left[y'^2 - y^2\right] dx; \; y(0) = 0, \; y\left(\frac{\pi}{2}\right) = 1.$$

OR

b) State the Brachistochrone problem as a variational problem stating the assumptions involved and then solve this problem.

11. a) State the isoperimetric problem. Solve this problem and find, its variational solution.

OR

b) Determine the extremals of the functional $v[y(x)] = \int_0^{\pi/2} \left(y''^2 - y^2 + x^2\right) dx$,

satisfying the conditions $y(0) = 1$, $y'(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$, $y'\left(\frac{\pi}{2}\right) = -1$.

12. a) Derive the differential equation of the free vibrations of a string using the variational principle.

OR

b) Derive the equation of vibrations of a rectilinear bar taking the x-axis along the axis of the bar in the equilibrium position. Deduce the case for the free vibrations of an elastic bar if the bar is homogeneous.

2,500

(1) GNLPP

(2) one constraint = L.g method

(3) Two constraint = L.g. H_0^B Avani

(3) K.T one constains ≤. ≥, case(i) $\lambda = 0$ case(ii) $\lambda \neq 0$

(9) Ayesha

(17)

Code No. 23/25/MS/4.3b/AOR

(4) K.T Two constains ≤ ≥

case(i) $\lambda_1 = 0, \lambda_2 = 0$

case(ii) $\lambda_1 = 0, \lambda_2 \neq 0$

case(iii) $\lambda_1 \neq 0, \lambda_2 \neq 0$

case(iv) $\lambda_1 \neq 0, \lambda_2 = 0$

(13)

FACULTY OF SCIENCE

M.Sc. (Mathematics) IV – SEMESTER

REGULAR/BACKLOG EXAMINATIONS, MAY 2014

ADVANCED OPERATIONS RESEARCH

PAPER – IIIb

Time: 3 hours] [Max. Marks: 70

Note: Answer all the question from Section – A and Section – B

Section – A

Answer any five of the following questions in not more than **ONE** page each: (5x4=20)

canonical form

1. Explain the following terms:
   i) Mixed strategies ii) Zero-sum game iii) optimal strategies iv) Value of the game
2. Define NLPP. Give two examples of NLPP.
3. What are the three time estimates used in PERT? Define the expected time.
4. Define QPP. Give two examples.
5. Explain the advantage of using the language multipliers method in solving NLPP.

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) Explain the method of solving a 2 × 2 game by:
   i) Arithmetic method ii) Algebraic method iii) Matrix method

(OR)

b) State dominance principle. Solve the game $\begin{bmatrix} 5 & -10 & 9 & 0 \\ 6 & 7 & 8 & 1 \\ 8 & 7 & 15 & 1 \\ 6 & 4 & -1 & 4 \end{bmatrix}$ A: $\frac{61}{19}$

7. a) Explain the procedures for forward pass and backward pass calculations in finding the critical path.

(OR)

b) For the following data, calculate the expected duration of each activity. Draw the network diagram. Using the CPM, obtain the minimum completion time of the project, after identifying the critical path.

Activity	Time in days		
	t_0 Optimistic	Most likely	t_P Pessimistic
1-2	2	5	14
1-3	9	t_m 12	15
2-4	5	14	17
3-4	2	5	8
4-5	6	6	12
3-5	8	17	20

8. a) Obtain the necessary and sufficient conditions for a GNLPP involving n variable and m constraints (m<n), using Lagrange multipliers method. Hence list out the conditions for maxima and minima.

(OR)

b) Use the method of Lagrangian multipliers to solve the following NLPP. Does the solution maximize or minimize the objective function?

Optimize: $Z = 2x_1^2 + x_2^2 + \ldots + 10x_1 + 8x_2 + 6x_3 - 100$

STC $g(x) = x_1 + x_2 \ldots = 20$

$x_1, x_2, x_3 \geq 0$

$x_1 = 5$

$x_2 = 11$

[PTO

(b) 40

9. a) ~~Explain Beale's method of solving a NLPP.~~

(OR)

b) Explain Wolfe's method of solving QPP. Use the above method to solve the following QPP.

Maximize $Z = 2x_1 + 3x_2 - 2x_1^2$

STC $x_1 + 4x_2 \leq 4$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

10. a) Use graphical method for solving the following game and find the value of the game.

Player – A	Player – B			
	B_1	B_2	B_3	B_4
A_1	2	2	3	-2
A_2	4	3	2	6

(OR)

b) Define NLPP. Solve the following NLPP using graphical method.

Minimize $Z = x_1^2 + x_2^2$

STC $x_1 + x_2 \geq 4$

$x_1 + x_2 \geq 5$

$x_1, x_2 \geq 0$

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Code No. 1843B

FACULTY OF SCIENCE

M.Sc. (Mathematics) IV – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017

ADVANCED OPERATIONS RESEARCH

PAPER – III b

Time: 3 hours] [Max. Marks: 70

Note: Answer all the question from Section – A and Section – B

Section – A

Answer any five of the following questions in not more than **ONE** page each: (5x4=20)

1. Explain maxi-min and mini-max principle in game theory.
2. Distinguish between CPM and PERT
3. Write down the canonical form of NLPP.
4. Write the steps of Beale's algorithm for QPP
5. What are the rules to be followed to represent a project by network diagram?

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) Solve the following Game graphically.

Player B

Player A	I	II	III	IV
I	2	2	3	-2
II	4	3	2	6

(OR)

b) Using dominance property solve the following game.

$$\begin{array}{cc} & \textit{Player B} \\ & \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \textit{Player A} \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array} & \begin{bmatrix} 1 & 7 & 3 & 4 \\ 5 & 6 & 4 & 5 \\ 7 & 2 & 0 & 3 \end{bmatrix} \end{array}$$

7. a) A project consists of a series of activities A,B,C,….H,I with the following relationships(W<X, Y means X and Y cannot start until W is completed; X,Y <W means W cannot start until both X and Y are completed) with this notation construct the network diagram having the following constraints: A<D,E; B,D<E; C<G; B<H; F,G<I. Find also the optimum time of completion of the project, when the time (in days) of completion of each activity is as follows.

Activity :	A	B	C	D	E	F	G	H	I
Time :	23	8	20	16	24	18	19	4	10

(OR)

b) Given the following project.

Activity	0-1	0-2	1-3	2-4	3-4	3-6	4-7	6-7
Duration (in weeks)	4	7	8	3	4	7	8	9

i) Draw the network diagram. Ii) Find critical path and project duration.

8. a) State and prove Kuhm-Tucker necessary and sufficient conditions in non-linear programming.

(OR)

b) Solve the NLPP using the method of Lagrangian multipliers

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Min $Z = 6x_1^2 + 5x_2^2$

STC $x_1 + 5x_2 = 3$ *and* $x_1, x_2 \geq 0$

9. a) Use Wolfe's method for solve the following.

Max $Z = 2x_1 + x_2 - x_1^2$

S.T.C $2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4$ *and* $x_1, x_2 \geq 0$

(OR)

b) Solve by Beale's method.

Minimize $Z = 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$

Subject to: $x_1 + x_2 \leq 2$ *and* $x_1, x_2 \geq 0$

10.a) Explain Arithmetic method for nxn games.

(OR)

b) Find the dimension of a rectangular parallelepiped with largest volume whose sides are parallel to the coordinate planes to be inscribed in the ellipsoid $g(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$. Using Lagrange's method.

--oOo--

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Code No.: 4786

FACULTY OF SCIENCE

M.Sc. IV Semester Examination, May/June 2010

MATHEMATICS / APPLIED MATHEMATICSS

Paper III (C)

(Advanced Operation Research)

Time : 3 Hours] [Max. Marks : 80

Answer **all** *questions.*

Section A – (Marks : $8 \times 4 = 32$)

1. Define Dominance Property in games with illustration.
2. Explain Minimax Principle of Game theory.
3. For a project network write the steps for finding the critical path in the forward pass calculations.
4. Distinguish between CPM and PERT
5. Explain about canonical form in NLPP.
6. Write sufficient condition for a general NLPP with one constraint with illustration.
7. Write down Kuhn–Tucker's condition for Q.P.
8. Explain the step of Wolfe's method for solving a QPP.

Section B – (Marks : $4 \times 12 = 48$)

9. (a) Solve the game graphically

$$A\overset{B}{\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & -3 \end{bmatrix}}$$

Or

(b) Solve the game using dominance property

	I	II	III	IV	V	VI
I	4	3	1	3	2	2
II	4	3	7	–5	1	2
III	4	[illegible]	4	–1	2	2
IV	4		3	–2	2	2

[P.T.O.

10. (a) Give the following project

Activity	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration (days)	2	8	10	6	3	3	7	5	2	8

(i) Draw the network diagram.

(ii) Find the critical path and project duration.

Or

(b) Project consists of A,B,C...... H,I activities with selection

A<D, A<E, B<F, D<F, C<G, E<H, F<I, G<I

Activities duration are

Activity	A	B	C	D	E	F	G	H	I
Time (Weeks)	8	10	8	10	16	17	18	14	9

Find project completion time.

11. (a) Solve the following NLPP.

Min $Z = x_1^2 + x_2^2 + x_3^2$

Subject to: $4x_1 + x_2 + 2x_3 = 14,\ x_1, x_2, x \geq 0$

Or

(b) Using Kuhn–Tucker Conditions solve.

Min $Z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to : $x_1 + 3x_2 \leq 6, 5x_1 + 2x_2 \leq 10,\ x_1, x_2 \geq 0$

12. (a) Solve by (simplified) Wolfe's Simplex method.

Max $Z = 2x_1 + 3x_2 - 3x_1^2$

Subject to : $x_1 + 4x_2 \leq 4,\ x_1 + x_2 \leq 2,\ x_1, x_2 \geq 0$

Or

(b) Using Beale's method solve:

Max $Z = 2x_1 + 3x_2 - x_1^2$

Subject to : $x_1 + 2x_2 \leq 4,\ x_1, x_2 \geq 0$

(f)

(22) (20)

34

Code No. : **1003**

FACULTY OF SCIENCE

M.Sc. IV Semester Examination, May 2011

MATHEMATICS/APPLIED MATHEMATICS

Paper – III (C) (403) : Advanced Operations Research

Time : 3 Hours] [Max. Marks : 80

Note : *Answer* ***all*** *questions.*

SECTION – A

(8×4=32 Marks)

1. What is game theory ? Explain briefly.
2. Explain maxi-min and mini-max principle used in game theory.
3. List out common errors present in drawing network diagrams.
4. Define (a) expected time (b) variance in relation to activities.
5. Find the maximum or minimum of the function

$$f(x) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 10.$$

6. Explain the formulation of non-linear programming problem.
7. Derive Kuhn-Tucker necessary conditions for an optimal solution to a quadratic programming problem.
8. Define general quadratic programing problem.

SECTION – B

(4×12=48 Marks)

9. a) Explain the principle of dominance and hence solve the following game.

Player B

$$\text{Player A} \quad \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} \text{I} & \text{II} & \text{III} \end{array} \\ \begin{bmatrix} 6 & 8 & 6 \\ 4 & 12 & 2 \end{bmatrix} \end{array}$$

OR

9

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b) Solve the following game by linear programming

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} -1 & 2 & 1 \\ 1 & -2 & 2 \\ 3 & 4 & -3 \end{bmatrix} \end{array} \quad \begin{pmatrix} 1 & 2 \\ 3 & -3 \end{pmatrix}$$

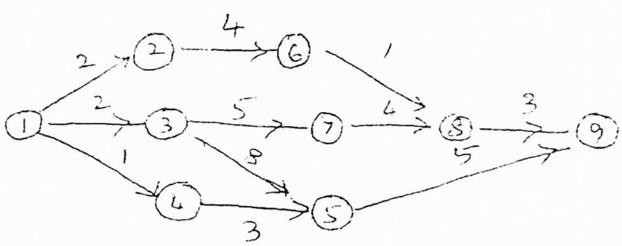
10. a) A project consists of a series of tasks labelled A, B, C, D, E, F, G, H, I with the following relationships (W < X, Y, means X and Y cannot start until W is completed ; X, Y < W means W can not start until both X and Y are completed) . With this notation, constrnct the network diagram having the following constraints :

A < D, E ; B, D < F ; C < G ; B < H ; F, G < I find also the optimum time of completion of the project, when the time (in days) of completion of each task is as follows :

Task :	A	B	C	D	E	F	G	H	I
Time :	23	8	20	16	24	18	19	4	10

OR

b) Find the critical path and calculate the slack time for each event for the following PERT diagram.



11. a) State and prove Kuhn-Tucker necessary and sufficient conditions in non-linear programming.

OR

b) Maximize $Z = x_1^2 + x_2^2 + x_3^2$, subject to the constraints $4x_1, + x_2 + 2x_3 = 14$, and $x_1, x_2, x_3 \geq 0$.

12. a) Apply Wolfe's method to solve the quadratic programming problem

Max $Z_x = 2x_1, + x_2 - x_1^2$, subject to $2x_1 + 3x_2 \leq 6$, $2x_1 + x_2 \leq 4$, and $x_1, x_2 \geq 0$.

OR

b) Use Beele's method for solving the quadratic programming problem

Max $Z_x = 4x_1 + 6x_2 - 2x_1^2 - 2x_1 x_2 - 2x_2^2$ subject to $x_1 + 2x_2 \leq 2$ and $x_1, x_2 \geq 0$.

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Code No. : 9492

FACULTY OF SCIENCE

M.Sc. IV Semester Examination, May/June 2012

MATHEMATICS/APPLIED MATHEMATICS

Paper – III (C) : Advanced Operation Research

Time : 3 Hours] [Max. Marks : 80

Note : *Answer **all** questions.*

SECTION – A (8×4=32 Marks)

1. Solve the game $A \overset{B}{\begin{bmatrix} 1 & 1 \\ 4 & -3 \end{bmatrix}}$.

2. Write a game as a LPP.

3. Explain briefly with examples two types of floats in networks-analysis.

4. Write the rules for drawing network diagram.

5. Write the Kuhn-Tucker conditions for Min $z = x_1^2 + x_2^2 + x_3^2$

   subject to $2x_1 + x_2 - x_3 \le 0$.

   $1 - x_1 \le 0$

   $2 - x_2 \le 0$

   $-x_3 \le 0$.

6. Find the maxima, minima for $f(x) = 2x_1^2 + 2x_2^2 + 2x_3^2 - 12x_1 - 8x_2 - 4x_3 + 2$.

7. Write the steps of graphical solution for NLP.

8. Explain the steps of Beak's method

(This paper contains 3 pages)

P.T.O.

(d) 37

Code No. : 9492

SECTION – B (4×12=48 Marks)

9. a) Solve the game graphically $A \overset{B}{\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix}}$.

OR

b) Solve the game by matrix method $A \overset{B}{\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}}$.

10. a) Given the following projects A, B, C, . . H, I and the relations A < D, E < F, D < F, E < H, E < I activity durations are

Activity	A	B	C	D	E	F	G	H	I
Time (Months)	1	6	2	7	8	5	4	9	3

Find the project time.

OR

b) Given the following project.

Activity	0-1	0-2	1-3	2-4	3-4	3-6	4-7	6-7
Duration (weeks)	4	7	8	3	4	7	8	9

i) Draw the network diagram.

ii) Find the critical path and project duration.

(19)

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Code No. : 9492

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11. a) Solve the following NLPP

$$\text{Max } z = 4x_1^2 + x_2^2 + 2x_3^2$$

subject to $x_1 + 2x_2 + x_3 = 16$

$x_1, x_2, x_3 \geq 0.$

OR

b) Using Kuhn-Tucker condition solve

$$\text{Max } z = 7x_1^2 - 6x_1 + 5x_2$$

subject to $x_1 + 2x_2 \leq 10$

$x_1 - 3x_2 \leq 9$ / $x_1, x_2 \geq 0.$

(4) cases.

(i) $\lambda_1 = \lambda_2 = 0$

(ii) $\lambda_1 = 0, \lambda_2 \neq 0$

(iii) $\lambda_1 \neq 0, \lambda_2 \neq 0$

(iv) $\lambda_1 \neq 0, \lambda_2 = 0$

12. a) Solve by Wolfe's method

$$\text{Max } z = 2x_1 + 5x_2 + x_1x_2 - x_1^2 - x_2^2$$

subject to $3x_1 - x_2 \leq 10$

$x_1, x_2 \geq 0.$

OR

b) Solve by Beak's method

$$\text{Max } z = 2x_1 + 2x_2 - 2x_2^2$$

subject to $x_1 + 4x_2 \leq 4$

$x_1, x_2 \geq 0.$

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Code No.: 5646

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FACULTY OF SCIENCE

M.Sc. IV Semester Examination, April/May 2009

MATHEMATICS/APPLIED MATHEMATICS

Paper III (C)

(Advanced Operations Research)

Special Note : In case of Old Batch students, marks scored would be converted to maximum marks of 100.

Time : 3 Hours] [Max. Marks : 80

Answer **all** *questions.*

Section A – (Marks : $8 \times 4 = 32$)

1. Define sadle point and the value of the games.

2. Explain Minimax principle.

3. Define total float and free float of an activity in project management network with examples.

4. Write the distinctions between PERT and CPM.

5. Define a general NLPP (problem) with an example.

6. Write Kuhn-Tucker conditions for a general NLPP with m ($< m$) constraints and illustrate.

7. Explain General Quadratic Programming Problem.

8. Write the steps of Beale's algorithm for QPP.

Section B – (Marks : $4 \times 12 = 48$)

9. (a) Solve the following game graphically

$$\begin{array}{cc} & B \\ & \begin{array}{cc} \mathrm{I} & \mathrm{II} \end{array} \\ A\ \begin{array}{c} I \\ II \\ III \end{array} & \begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix} \end{array}$$

Or

[P.T.O.

L{y(x)}

(K)

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(b) Solve the game using dominance property.

$$
\begin{array}{cc}
 & \begin{array}{ccccc} & & B & & \\ I & II & III & IV & V \end{array} \\
A\ \begin{array}{c} I \\ II \\ III \end{array} & \begin{bmatrix} 4 & 4 & 2 & -4 & -6 \\ 8 & 6 & 8 & -4 & 0 \\ 10 & 2 & 4 & 10 & 12 \end{bmatrix}
\end{array}
$$

10. (a) The activity of a project has the following time schedule. Draw the project network and find the critical path.

Activity	1–2	1–3	1–4	2–5	3–6	3–7	4–6	5–8	6–9	7–8	8–9
Duration days	2	2	1	4	8	5	3	1	5	4	3

Or

(b) Draw project network and find critical path.

Activity	1–2	1–3	2–4	3–4	3–5	6–9	5–6	5–7	6–8	7–8	8–9	9–10
Duration weeks	4	1	1	1	6	5	4	8	1	2	1	9

11. (a) Solve the following NLPP by the method of Lagrangian multiplies.

Min t = $6x_1^2 + 5x_2^2$

subject to :

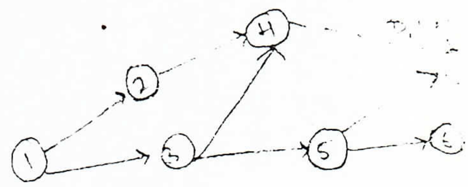
$x_1 + 5x_2 = 3$, $x_1, x_2 \geq 0$.

Or

(b) Using Kuhn-Tucker condition solve :

Min t = $x_1^2 + x_2^2 + x_3^2$ subject to : $2x_1 + x_2 \leq 5$,

$x_1 + x_3 \leq 2$, $x_1, x_2, x_3 \geq 0$.



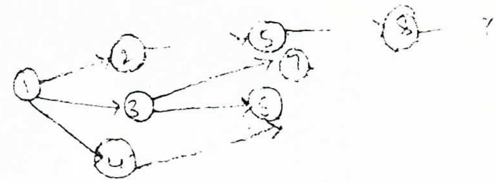
12. (a) Solve by Wolf's modified simplex method

Max Z = $2x_1 + x_2 - x_1^2$

subject to : $2x_1 + 3x_2 \leq 6$, $2x_1 + x_2 \leq 4$, $x_1, x_2 \geq 0$.

Or

(b) Using Beale's method solve:



Max Z = $2x_1 + 3x_2 - 2x_2^2$ subject to :

$x_1 + 3x_2 \leq 4$, $x_1 + x_2 \leq 2$, $x_1, x_2 \geq 0$.

K-T N.C.

K-T N.C. (MAX)

ii) If f(z) is analytic in an annulus $r < |z - z_0| < R$, then show that we can find a Laurent's series $\sum_{n=-\infty}^{\infty} A_n (z-z_0)^n$ such that

$$f(z) = \sum_{n=-\infty}^{\infty} A_n (z-z_0)^n$$

OR

b) i) Show that the infinite product $\prod_{n=1}^{\infty}(1+a_n)$ with $1 + a_n \neq 0$ converges if and only if the series $\sum_{n=1}^{\infty} \log(1+a_n)$ converges. (Here log z represents the principal branch of the logarithm).

ii) Show that

$$2^{2z-1}\,\Gamma(z).\Gamma\left(z+\frac{1}{2}\right) = \sqrt{\pi}\,\Gamma(2z)$$

12. a) i) Obtain Poisson - Jensen's formula.

ii) For $\sigma = \text{Re } s > 1$, show that $\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty}(1-p_n^{-s})$.

OR

b) State and prove Hadamard's theorem.

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Code No. Code No. 1844

FACULTY OF SCIENCE

M.Sc. (Mathematics) IV – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017

BANACH ALGEBRA

PAPER – IV

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer any five of the following questions in not more than **ONE** page each: (5x4=20)

1. In a normal algebra, prove that the multiplication is jointly continuous.
2. Define a topological divisor of zero (TDZ) and show that every TDZ is singular.
3. If A and B are C^*- algebra with units and if φ: A→B is a *-homomorphism such that φ(1)=1, then show that $\| \varphi(a) \| \leq \| a \|$ for all a∈A.
4. Let H be a Hilbert space, T∈L(H). If $\| T \| \leq 1$ then prove that for any complex polynomial f $\| f(T) \| \leq \| f \|_{\Delta_1}$ where $\Delta_1 = \{\lambda \in C \,/\, |\lambda| \leq 1\}$.
5. Let A be a C^*-algebra with unity and ∝ be any element of A. Show that there exists a normalized state f on A. Such that $f(\propto^* \propto) = \|\propto^* \propto\|$.

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) If A is an associative algebra over c, with unity element u, and if A has a Banach space norm x→||x|| such that (x,y)|→xy is separately continuous in each factor, then show that there exists an equivalent norm x→| x | on A such that (A, ||) is a Banach algebra and such that | u |=1.

(OR)

b) State and prove Gelfand formula for the spectral radius.

7. a) If x∈A, σ(x)≠φ, then show σ(f(x))=f(σ(x)) for all f∈C(t, σ(x)). Here A is an associative algebra.

(OR)

b) State and prove spectral mapping theorem for rational functional calculus.

8. a) State and prove commutative Gelfand newmark theorem.

(OR)

b) i)If A and B are C^*-algebra with unity land if ϕ: A→B is a unibal algebra homomorphism such that $\|\phi(a)\| \leq \|a\|$ for all a∈A, then show that ϕ is a * -homomorphism.

ii) Show that in a C^*-algebra with unity the sum of positive elements in positive.

9. a) If H is a Hilbert space and T is an operator on H then show that T is unity if and only if the unit circle $\left\{\lambda \in C \,/_{\lambda^* \lambda = 1}\right\}$ is a spectral set for T

(OR)

b) If H is a Hilbert space and T is an operator on H such that $\| T \| \leq 1$ and $f \in C(t; \Delta_1)$, then prove that there exist a sequence of complex polynomials for such that $\| f_n(T) - f(T) \| \rightarrow 0$.

10. a) If E is a Banach space and T∈L(E) then prove that the following conditions on T are equivalent.

i) T is subjective ii) T^1 is bounded below

(OR)

b) Let A be a C^*-algebra with unity. Show that there exists a Hilbert space H and an isometrical unital *-representation ϕ:A→Z(H).

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Code No.: 5650

FACULTY OF SCIENCE

M.Sc. IV Semester Examination, April/May 2009

MATHEMATICS/APPLIED MATHEMATICS

Paper V (A) (405)

(Calculus of Variations)

Special Note: In case of Old Batch students, marks scored would be converted to maximum marks of 100.

Time : 3 Hours] [Max. Marks : 80

Answer **all** *questions.*

Section A – (Marks : 8 × 4 = 32)

1. Define a linear functional and variation of a functional. Give one example.

2. State and prove the fundamental Lemma of calculus of variations.

3. Find the curve joining the origin and the point $A(1, 1)$ whose rotation about the axis of abscissa generates a surface of minimum area.

4. Show that the extremal of the functional $V[y, x] = \int_{x_0}^{x_1} \frac{1}{y}\sqrt{1+y'^2}\,dx$; represents the family of circles centred on x-axis.

5. Solve the variational problem $V[y, x] = \int_0^1 y \cdot y'^3 dx$; $y(0) = 0$, $y(1) = 1$.

6. Write the ostrograd sky equation for the functional
$$v[z(x, y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2\right] dx\,dy.$$

7. Find the extremals of the functional $V[y(x)] = \int_0^{\pi/2} (y''^2 - y^2 + x^2)\,dx$.

8. Derive the equations of motion of a projectile in space using Hamiltons equations.

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(b) 48

Section B – (Marks : 4 × 12 = 48)

9. (a) (i) Derive the necessary condition for the functional $V[y(x)] = \int_a^b F(x, y, y')\, dx$ with the boundary conditions $y(a) = y_a$, $y(b) = y_b$ to have an extremum.

(ii) Prove that the shortest distance between two points in a plane is a straight line.

Or

(b) (i) On which curve the functional $\int_0^{\pi/2} [(y')^2 - y^2 + 2xy]\, dx$ with $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$ be extremized.

(ii) Does the functional $V[y(x)] = \int_0^1 y\sqrt{1 + y'^2}\, dx$ with boundary conditions $y(0) = y(1) = \frac{1}{\sqrt{2}}$ possess an extremum? Justify your claim.

10. (a) Explain the Brachistochrone problem and find a variational solution to it.

Or

(b) Find a curve with specified boundary points whose rotation about the axis of abscissa generates a surface of minimum area.

11. (a) State the isoperimetric problem and obtain its solution using the principle of variational calculus.

Or

(b) Determine the extremal of the functional $v[y(x)] = \int_{-l}^{l} \left(\frac{1}{2}\mu y''^2 + \rho y\right) dx$ that satisfies the boundary conditions $y(-l) = 0$, $y'(-l) = 0$, $y(l) = 0$, $y'(l) = 0$.

12. (a) Derive the Euler-Poisson equation.

Or

(b) Derive the differential equation of the free-vibration of a string using variational principle.

ⓔ (29) (39)

Code No. : 1007 (13)

41 (19)

FACULTY OF SCIENCE
M.Sc. IV Semester Examination, May 2011
MATHEMATICS/APPLIED MATHEMATICS
Paper – V (A) : Calculus of Variation

Time : 3 Hours] [Max. Marks : 80

*Note : Answer **all** questions.*

SECTION – A **(8×4=32 Marks)**

1. Distinguish between a function and a functional. Define a linear functional and a geodesic.

2. Explain the concept of a strong extremum and weak extremum of functionals. Show that if a strong maximum (or minimum) is attained on a curve then a weak one is attained. Is the converse held in general ? Justify your claim.

3. Find the extremals of the functional $v[y(x), z(x)] = \int_0^{\pi/2} \left[y'^2 + z'^2 + 2yz\right] dx$ subject to $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 1$.

4. Test for an extremum the functional $v[y(x)] = \int_0^1 (xy + y^2 - 2y^2 y')dx$, $y(0) = 1$ and $y(1) = 2$.

5. Find the Ostrogradsky equation for the functional

$$S[z(x, y)] = \iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\, dy$$ and hence investigate for its minimum.

(8)

42

Code No. : 16

6. Show that if the integrand of the functional $v[x(t), y(t)] = \int_{t_0}^{t_1} \phi(t, x(t), y(t), \dot{x}(t), \dot{y}(t))\, dt$ does not contain t explicitly and is a homogeneous function of first degree in $\dot{x}$ and $\dot{y}$ then that functional does not depend on its parametric representation.

7. Using the variational principle, find the differential equations of motion of the system with n particles of masses m_i (i = 1, 2, ..., n) and coordinates (x_i, y_i, z_i) and acted upon by forces $\overline{F}_i$ that possess the potential $-U$ satisfying

$$F_{ix} = -\frac{\partial U}{\partial x_i},\ F_{iy} = -\frac{\partial U}{\partial y_i} \text{ and } F_{iz} = -\frac{\partial U}{\partial z_i}.$$

8. Derive the equations of motion of a projectile in space using the Hamiltons equations.

SECTION – B (4×12=48 Marks)

9. a) i) State and prove the fundamental Lemma of calculus of variation.

ii) Determine the curve on which the functional $v[y(x)] = \int_0^{\pi/2} \left[(y')^2 - y^2\right] dx$, $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 1$ can be extremized.

OR

b) i) Derive the Eulers equation for finding the extremals of the functional

$$v[y(x)] = \int_{x_0}^{x_1} F(x, y, y')\, dx.$$

ii) Show that the extremals of the functional $l[y(x)] = \int_{x_0}^{x_1} \sqrt{1+y'^2}\, dx$ are the straight lines.

20

Ayesha.

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FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) IV - SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2012

CALCULUS OF VARIATIONS

PAPER - V

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

(5x4=20)

Answer the following questions in not more than ONE page each:

1. The differential operator and variational operator commutes is $\frac{d}{d^x}(\delta y) = \delta\frac{dy}{dx}$.
2. Show that straight line is the shortest distance between two given lines in a plane.
3. Find the plane curve with fixed perimeter of max area (or) determine the closed convex curve of given circumstances which encloses max area.
4. Derive the Hamilton's equation of motion.
5. Determine the externals of the functional $V\{y(x)\} = \int_0^{\pi/2}(y'' \ldots)dx$ with the boundary conditions $y(0)=1$; $y'(0)=0$, $y(\pi/2) = 0$, $y'(\pi/2) = -1$.

Section – B

(5x10=50)

Answer the following questions in not more than FOUR pages each.

6. a) i) On what curve can the function ... boundary conditions $y(0)=0$, $y(1)=1$

ii) Find the extremale of the function ... $1 + (y')^2 dx$

(OR)

b) Derive Euler Lagranges equation.

7. a) State and prove Branchisto-Crone problem.

(OR)

b) i) Find the extremals of the functional $V\{y(x), z(x)\} = \int_0^{\pi/2}(y'^2 + z'^2 + 2yz)dx$ with boundary values $y(0)=0$, $y(\pi/2) = 1$, $z(0)=0$ and $z(\pi/2) = 1$.

ii) Find extremal of the functional $V\{y(x), z(x)\} = \int_{x_0}^{x_1}(2yz - 2y^2 - y'^2 - z'^2)dx$

8. a) Find the solution of the isoperimetric problem $I\{y(x)\} = \int_0^1 (y'^2 + x^2)dx$ with boundary conditions $y(0) = 0$, $y(1) = 0$ which satisfy constrains condition $\int_0^1 y^2 dx = 2$.

(OR)

b) i) ... $V[z(x, y)] = \iint_D \left[\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\right] dxdy$

ii) ... variational problem ...

(ii) Derive variation problem in ... closed ... form

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Code No.2127/5/M/3/4.5/R[illegible]

9. a) Derive time Hamilton's principle.

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(OR)

b) i) Find the extremum of the functional $V\{y(x)\} = \int_0^1 (1 + y''^2)dx$; $y(0) = 0, y(1) = 1$, $y'(0) = 0$ and $y'(1) = 1$. 0345678 7756789

ii) Determine the exteremals of the functional $V\{y(x)\} = \int_0^{\pi/2} (y''^2 - y^2 + x^2)dx$ with the boundary conditions $y(0) = 1, y'(0) = 0; y(\pi/2) = 0$ and $y'(\pi/2) = -1$. 78 8 8 8

10. a) State and prove the problem of fr[illegible]odisc.

(OR)

b) Determine the curve passing through $(x_1,y_1), (x_2,y_2)$ which when rotate about [illegible]xis given a minimum surface area.

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Code No. Code No. 1845

FACULTY OF SCIENCE

M.Sc. (Mathematics) IV – SEMESTER REGULAR/BACKLOG EXAMINATIONS, APRIL 2015

45

CALCULUS OF VARIATIONS

PAPER – V

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer any five of the following questions in not more than ONE page each: (5x4=20)

1. Define strong and week variations.
2. What is the problem of the brachistochrone?
3. Find the extremals of the isoperimetric problem $V[y(x)] = \int_0^1 (y'^2 + x^2)dx$ given that $\int_0^1 y^2 dx = 2$; $y(0)=0$; $y(1)=0$
4. Prove that the shortest distance between two points in a plane is a straight line.
5. State and prove the Hamilton's principle.

Section – B

Answer the following questions in not more than FOUR pages each: (5x10=50)

6\. a) On what curves can the functional $V[y(x)] = \int_0^1 [(y')^2 + 12xy]dx$, $y(0) = 0$, $y(1) = 1$ be extremized

(OR)

b) State and prove the fundamental lemma of calculus of variation.

7\. a) Find the curve y= y(x) of given length l for which the areas of the curvilinear trapezoid CABD.

(OR)

b) i) Write the Ostrogradsky equation for the functional $V =$ [illegible]

ii) Determine the Biharmonic equation for the functional $V = \iint_D [(\frac{\partial^2 z}{\partial x^2})^2 + (\frac{\partial^2 z}{\partial y^2})^2 + 2(\frac{\partial^2 z}{\partial x \partial y})^2]dxdy$

8\. a) Derive the Euler-Ostorgradsky equation to the functional $V[z(x,y)] = \iint_D F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})dxdy$.

(OR)

b) Show that the sphere is the solid figure of revolution, which for a given surface area, has maximum volume.

9\. a) Derive the differential equation free vibration of a string.

(OR)

b) Derive the equation of a vibrating linear rod.

10\. a) State and prove Lagrange equation of motion.

(OR)

b) Find the extremal of the functional $V[y(x)] = \int_0^1 (1 + y''^2)dx$, $y(0)=0$, $y'(0) = 1$, $y(1)=1$, $y^1(1) = 1$.

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(15)

Code No. 23/25/MS/4.5/EV

FACULTY OF SCIENCE
M.Sc. (Mathematics) IV – SEMESTER
REGULAR/BACKLOG EXAMINATIONS, MAY 2014

CALCULUS OF VARIATIONS

PAPER – V

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer any five of the following questions in not more than **ONE** page each: (5x4=20)

1. Define functional and linear functional.
2. What is the problem of the brachistochrone?
3. Find the extremals of the function $V[y(x)] = \int_{x_0}^{x_1}[16y^2 - y''^2 + x^2]dx$.
4. State Hamiltonian's principle.
5. Derive the differential equation of motion of simple pendulum using Lagrange's equation.

Section – B

Answer the following questions in not more than FOUR pages each: (5x10=50)

6. a) Derive Euler's equation for the functional $V[y(x)] = \int_a^b F(x, y, y', y'')dx$ with the conditions $y(a) = \alpha, y'(a) = \beta, y(b) = r, y'(b) = 8$

(OR)

b) i) Distinguish between strong and weak variation with suitable examples.

ii) Find the extremal of the functional $V[y(x)] = \int_0^1 (1 + y''^2)dx$ subject to $y(0) = 0$, $y'(0) = 1$, $y(1) = 1$, $y'(1) = 1$.

$C_1 = 6$ $C_2 = 4$.

7. a) Using the variational principle, find the curve connecting two given points sliding from A and B, that is traversed by a particle sliding from A to B in the shortest time.

(OR)

b) Find the curve passing through the points (x, y), (x_2, y_2) which when rotated about x-axis gives minimum surface area.

8. a) Derive the Euler-Ostorgradsky equation for the functional $V[z(x, y)] = \iint_D F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})dxdy$.

(OR)

b) Show that the sphere is the solid figure of revolution, which for a given surface area, has maximum volume.

9. a) Derive the Euler-Poisson equation.

(OR)

b) Derive the differential equation of vibrating linear rod.

10. a) State and prove Hamilton's canonical equations.

(OR)

b) i) State and prove the fundamental lemma of calculus of variation.

ii) Prove that the shortest distance between two points in a plane is a straight line.

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(5)

Code No. Code No. 1845

FACULTY OF SCIENCE

M.Sc. (Mathematics) IV – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017

CALCULUS OF VARIATIONS

PAPER – V

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer any five of the following questions in not more than ONE page each: (5x4=20)

1. Show that the functional $L\{y(x)\} = \int_0^1 (y(x) + y^1(x))dx$ is linear.
2. Find the extremal of the functional $V\{x(t), y(t)\} = \int_0^1 (y^{1^2} + x^{1^2} + 2y)dt$ where boundary condition are
   $x(0)=0;\ y(0)=1$
   $x(1)=1;\ y(1)=\frac{3}{2}$
3. Find the solution of the isoperimetric problem $I\{y(x)\} = \int_0^1 (y'^2 + x^2)dx$ with boundary conditions $y(0)=0;\ y(1) = 0$ which satisfies constrains condition $\int_0^1 y^2 dx = 2$.
4. Find the exthemum of $V\{z(x)\} = \int_{x_0}^{x_1} y'^2 dx$ given $\int_{x_0}^{x_1} y dx = a$.
5. Show that straight line is the shortest distance between two given lines in a plane.

Section – B

Answer the following questions in not more than FOUR pages each: (5x10=50)

6. a) Derive the Euler legranges equation or derivation of Euler integral.

(OR)

b) (i) State and prove the fundamental Lemma of calculus of variation.

(ii) On what curves can the functional $V[y(x)]\int_0^{\pi/2}((y^1)^2 - y^2)\,dx$ with the condition $y(0)=0$ $y(\pi/2)=1$ is extrimized.

7. a) State and prove the problem of Geodisc.

(OR)

b) i) To find the curve when fixed boundary such that its rotation and revolution of a minimum surface of aria.

ii) Find the extremals of the functions $V\{y(x), z(x)\} = \int_{x_0}^{x_1} (2yz - 2y^2 - y^{1^2} - z^{1^2})dx$

8. a) Find plane curve which fixed parameter of maximum area.

(OR)

b) (i) Solve $V[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy.$

(ii) Solve $\iint_D \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)^2 dxdy.$

9. a) (i) $V\{y(x)\} = \int_{-l}^{l} \left[\frac{1}{2}\mu y^{11^2} + \rho y\right] dx$ that satisfy the boundary condition $y(-l)=0;\ y(l)=0,\ y^1(-l)=0$ and $y^1(l)=0$.

(ii) Find the extremum of the functional $V\{y(x)\} = \int_0^1 (1 + y^{11^2})\,dx;\ y(0)=1,\ y(1)=1,\ y^1(0)=0,$ $y^1(1)=0$.

(OR)

b) Derive the Euler's Poisson's equation.

10. a) State and prove Branchisto-crone problem.

(OR)

b) State and prove Hamilton's principle.

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