Code: 1821

Faculty of Sciences

M.Sc (Mathematics) II-Semester Regular Examinations, Aug-2023

Paper- I: ADVANCED ALGEBRA

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. Let $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1} + x^n \in Z[x]$ be a monic polynomial. If f(x) has a root $a \in \emptyset$ then prove that $a \in Z$ and a/a_0.
2. Let P be a prime then prove that $f(x) = x^p - 1 \in Q(x)$ has splitting field $Q(\propto)$ where $\propto \neq 1$ and $\propto^p = 1$.
3. Suppose H is a subgroup of $G\left(\frac{E}{F}\right)$ then prove that $F \subset E_H \subset E$.
4. Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is solvable by radicals over $\emptyset$.
5. Show that $x^3 - x - 1 \in \emptyset(x)$ is irreducible over $\emptyset$.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) let $F \subseteq E \subseteq K$ be fields. If $[K:E] \propto \infty$ and $[E:F] \propto \infty$ then show that
   (i) $[K:F] \propto \infty$ **(ii)** $[K:F] = [K:E][E:F]$

   (OR)

   (b) Let p(x) be an irreducible polynomial in F[x] and let u be a root of p(x) in an extension E of F. Let deg p(x)=n. If F(u) is the subfield of E generated by F and u, then prove that $\{1, u, u^2, \ldots, u^{n-1}\}$ forms a basis of F(u) over F.

7. (a) Find the degree of the extension of the splitting field of $x^3 - 2 \in \emptyset(x)$

   (OR)

   (b) If E is a finite separable extension of a field F, then prove that E is a simple extension of F.

8. (a) State and prove the fundamental theorem of Algebra.

   (OR)

   (b) State and prove the fundamental theorem of Galois Theory.

9. (a) Let E be a finite extension of F. Suppose $f: G \to E^*$, $E^* = E - (0)$ has the property that $f(\sigma\eta) = \sigma(f(\eta) f(\sigma)$ for all $\sigma \in G$ where $G = G(\frac{E}{F})$

   (OR)

   (b) If F contains a primitive n^{th} root of unity then prove that the following are equivalent.
   (i) E is a finite cyclic extension of degree n over F.
   (ii) E is the splitting field of an irreducible polynomial $x^n - b \in F(x)$.

10. (a) (i) If E is a finite extension of F the prove that E is an algebraic extension of F.
   (ii) If E is an extension of F and $u \in E$ is algebraic over F. Then prove that E is an algebraic extension.

   (OR)

   (b) (i) Let U be a finite subgroup of the multiplicative group F{0}, where F is a field then prove that U is cyclic group
   (ii) Complete $\emptyset_b(x)$

*****

Code:1822

Faculty of Sciences

M.Sc (Mathematics) II-Semester Regular Examinations, Aug-2023

Paper- II: ADVANCE REAL ANALYSIS

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. Let A be any subset of R with $m^*(A) < \infty$. Prove that given any $\in > 0$ then exists an open set O such that $A \subset 0$ and $m^*(O) < M^*(A) + \in$.
2. Define Lebesgue integral of a bounded measurable function. Suppose f, g are bounded measurable functions defined on a measurable Set E of finite measure and if f=g a.e prove that $\int_E f = \int_E g$.
3. Suppose f,g are functions of bounded variation m[a,b]. Prove that f + g is also a finte on a bounded variation on [a,b] also prove that $T_a^b(f+g) \leq T_a^b(f) + T_a^b(g)$.
4. Suppose A is a linear operator in R^n. Prove that A is invertible if and only if $\det|A| \neq 0$.
5. Suppose E is measurable subset of R. Prove that $E + \propto$ is measurable for any $\propto \in R$. Also prove that $m(E+\propto) = m(E)$.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) State and prove Littlewood third principle.

(OR)

(b) Prove that every Borel set is measurable.

7. (a) State and prove Lebesque dominated convergence theorem .

(OR)

(b) (i) State and prove Fatous Lemma for a sequence of non-negative measurable function.

(ii) State and prove monotone Convergence theorem for non-negative measurable function.

8. (a) If f is a bounded measurable function on [a,b] and if | $F(x) = \int_a^x f(t)dt + F(a)$ then prove that F=f almost most every where on[a,b].

(OR)

(b) State and prove Vital Convering lemma.

9. (a)(i)If [A] and [B] are n by n matrices then prove that det ([B][A]) = det [B] det[A].

(ii)Prove that a linear operator A one R^n is invertible if and only if det[A]≠0.

(OR)

(b) State and prove inverse function theorem.

10. (a) State and prove Minkowski's inequality.

(OR)

(b) If X is a normal Linear space then prove that X is complete if and only if every absolutely summable series in X is summable in X.

*****

Code:2823/R

Faculty of Sciences

M.Sc (Mathematics) II-Semester Regular Examinations, Aug-2023

Paper- III: FUNCTIONAL ANALYSIS

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. Prove that every finite dimensional subspace Y of a normed space X is closed in X.
2. Prove that the space l^p with $p \neq 2$ is not a Hillbert space.
3. Let M be a subset of an inner product space X. If M is total in X then prove that $x \perp M \Rightarrow x = 0$.
4. For every fixed X in a normed space X, define a functional g_x $on\ X'$ by $g_x(f) = f(x)$ prove that $||g_x|| = ||x||$.
5. Let T be a bounded linear operator on a Hillbert space H, then prove that
   (i) If T is self-adjoint <Tx,x> is real for all $x \in H$.
   (ii) If H is complex and <Tx,x> is real for all $x \in H$, the operator T is self-adjoint.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) Prove that every normed space with schauder basis is separable.

(OR)

(b) Let $\{x_1, x_2, \ldots. x_n\}$ be a linearly independent set of vectors in a normed space X. Then prove that there is number c>0 such that for every choice of scalars $\propto_1, \propto_2 - - \propto_n$ we have $||\propto_1 x_1 + \propto_2 x_2 + \propto_n x_n|| \geq (|\propto_1| + |\propto_2| + \cdots + |\propto_n|)$, $C > 0$

7. (a) State and prove Schwarz inequality and triangle inequality.

(OR)

(b) (i) Let Y be a complete subspace of an inner product space X and $x \in X$ fixed. Then prove that Z=x-y is orthogonal to Y.
(ii) If Y is a closed subspace of a Hillbert space H. Then prove that Y=Y''

8. (a) State and prove Riesz representation theorem for Sesquilinear form.

(OR)

(b) Let H be a Hillbert space. If H contains an orthonormal sequence which is total in H, then prove that H is separable.

9. (a) State and prove closed graph theorem.

(OR)

(b) State and prove Generalized Haln- Branch theorem.

10. (a) Prove that two Hillbert spaces H and $\tilde{H}$, both real or both complex are isomorphic if and only if they have the same Hillbert dimension.

(OR)

(b) State and prove Baire's Category theorem.

*****

Code: 2824/R

Faculty of Sciences

M.Sc (Mathematics) II-Semester Regular Examinations, Aug-2023

Paper- IV: THEORY OF ORDINARY DIFFERENTIAL EQUATION

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each **5x4=20 M**

1. Define a Linear independence and dependence of the function defined on an interval I
2. Prove that there exists three linearly independent solutions of the third order equation $x''' + b_1(t)x'' + b_2(t)x' + b_3(t)x = 0, t \in I$ where b_1, b_2 *and* b_3 are continuous functions defined on an internal I.
3. Find e^{At} when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.
4. Find the Lipschitz constant and bound for $f(t,x) = x^2$ in the region $R = \{(t,s): |t-1| \leq 1, |x+1| \leq 2\}$
5. State contraction principle and also prove that contraction mapping is continuous.

Section-B

II. Answer the following questions in not more than FOUR pages each **5x10=50 M**

6. (a) Let $\emptyset_1, \emptyset_2, \ldots..\emptyset_n$ be 'n' linearly independent solution of the equation $L(x) = x^{(n)} + b_1(t)x^{(n-1)} + \cdots + b_n(t)x = 0,\ t \in I$ let the real or complex valued function to be defined and continuous on I. Further assume that $w(t) = w(\emptyset_1, \emptyset_2, \ldots..\emptyset_n)$ and w(t) denote the determinant w(t) and Kth column replaced by n elements 0,0 .....1 then prove that particular solution $l(x) = x^{(n)} + b_1(t)x^{(n-1)} + \cdots + b_n(t)x = h(t)$ is $x_p(t) = \sum_{k=1}^{n} \emptyset_k(t) \int_{t_0}^{t_1} \frac{w_k(s)h(s)}{w(s)} ds,\ t \in I$.

   (OR)

   (b) State and prove the Abel's formula.

7. (a) State and prove existence and uniqueness theorem for a homogeneous linear differential system $x^1 = A(t)x,\ x(t_0)\ ;\quad t, t_0 \in I$

   (OR)

   (b) Determine the fundamental matrix from the system $X^1 = Ax$ where $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 5 \end{bmatrix}$.

8. (a) State and prove Grownwall inequality and hence using it, prove that f(t)=0 for $t \geq t_0\ if\ f(t) \leq k \int_{t_0}^{t} f(s)\, ds$ where f and k are as defined in Grownwall inequality.

   (OR)

   (b) Find first three successive approximation for the solution of IVP $x^1 = \frac{x}{1+x^2}, x(0) = 1$

9. (a) State and prove Bihari's inequality.

   (OR)

   (b) Define equi-continuous. Let the function f(t,x) be continuous and bounded an the infinite strip $S = \{(t,x);\ t_0 \leq t < t_0 + h, |x| < \infty\ (h > 0)\} in\ D.\ Then\ the\ IVP\ \ x^1 = f(t,x),$ $x(t_0) = x_0$ has atleast one solution x(t) existing on the interval I=$[t_0, t_0 + h\,]$.

10. (a) Solve $x''' + 6x'' + 11x^1 + 6x = 0 :\ -\infty < t < \infty$.

    (OR)

    (b)Prove that the set of all solutions of the system$X^1 = A(t).X$ on I forms an n-dimensional vector space over the field of complex numbers.

*****

Code: 2825/R

Faculty of Sciences

M.Sc (Mathematics) II-Semester Regular Examinations, Aug-2023

Paper- V: DESCRETE MATHEMATICS

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each **(5x4=20 M)**

1. Define a poset and dual of a poset, prove that the dual of a poset is also poset.
2. Minimize the Boolean expression$E(x, y, z) = (X' \oplus y) \oplus y'$.
3. Prove that there is an even number of vertices of odd degree in any graph.
4. Define tree, directed tree, rooted tree and five suitable examples.
5. Prove that K_s is the complete graph with five vertices in non-planar.

Section-B

II. Answer the following questions in not more than FOUR pages each **(5x10=50 M)**

6. (a) let $(L, \leq)$ be a lattice. For any $a, b, c \in L$ the following hold

    (i) $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$ (ii) $a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$

    (iii) $a * (b \oplus c) \geq (a * b) \oplus (a * c)$

    (OR)

    (b) (i) Define a distribution lattice. Is every Lattice distributive?

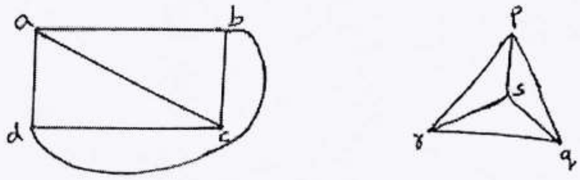
    (ii) Prove that every chain is a distribution Lattice.

7. (a) Define a Boolean expression. Show that the Boolean expressions $(x \oplus y) * (x' \oplus z) * (y \oplus z) and\ (x * z) \oplus (x' * y) \oplus (y * z)$ are equivalent. Obtain their sum of product canonical form.

    (OR)

    (b) State and prove the Stone's representation theorem for finite Boolean algebra.

8. (a) (i) Prove that the following graphs are isomorphic.



    (ii) How many isomorphisms can you find between them?

    (OR)

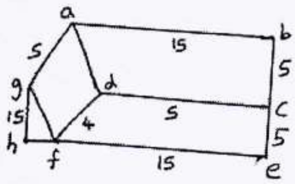
    (b) (i) State and prove Euler's formula for connected planner graph.

9. (a) (i) Prove that a simple non-directed graph G is a tree iff G is connected and containing no cycles.

(ii) Prove that a tree with two or more vertices has at least two leaves.

(OR)

(b) (i) Determine a minimal spanning tree for the following weighted connected graph



(ii) Define a cut set in a connected graph. Prove that a cut-set and any spanning tree in a connected graph have at least one edge in common .

10.(a) (i) Prove that every chain is a distributive Lattice.

(ii) Prove that the Demorgan's Law hold in a complemented distributive Lattice.

(OR)

(b)Let $(L,\leq)$ be a lattice for $a,b,c \in L$ prove that the following properties

(i) $b \leq c \Longrightarrow \begin{Bmatrix} a*b \leq a*c \\ a\oplus b \leq a\oplus c \end{Bmatrix}$ (ii) $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a\oplus b = b$

*****

Code:

Faculty of Sciences

M.Sc (Mathematics) I-Semester ~~Regular~~ B.Log Examinations, ~~March~~ Aug-2023

Paper- I: ALGEBRA

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. Define G-Set and give an example.
2. Show that any group of order 119 in cyclic.
3. Prove that the kernel of Homomorphism of a ring R into another ring R is an ideal of R
4. State and prove Schur's lemma.
5. Show that an group $\frac{Z}{(10)}$ is a direct sum of $H = \{\bar{0}, \bar{5}\}$ and $K = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}\}$

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) State and prove Jordan- Holder's theorem.

(OR)

(b) If G is a nilpotent group then prove that every subgroup of G and every homomorphic image of G are nilpotent .

7. (a) State and prove the first Sylow theorem.

(OR)

(b) Let A be a finite abelian group then prove that there exists a unique list of integers $m_1, m_2 \ldots\ldots\ldots . m_k$ $(all > 1)$ such that $|A| = m_1, m_2 \ldots\ldots\ldots . m_k$, $m_1|m_2| \ldots\ldots\ldots . |m_k$ and $A = c_1 \oplus c_2 \oplus \ldots\ldots\ldots . \oplus c_k$ where $c_1, c_2 \ldots\ldots\ldots . c_k$ are cyclic subgroups of A of order $m_1, m_2 \ldots\ldots\ldots . m_k$ respectively.

8. (a) Let R be a non zero ring with unity 1 and I is an ideal of R such that $I \neq R$ then show that there exists a maximal ideal M of R such that $I \subseteq M$.

(OR)

(b) Prove that every PID is a UFD, but a UFD is not necessarily a PID.

9. (a) Let R be a commutative ring with regular element let 'S' be the set of all regular elements in R then prove that R_s has the following properties.
   1) R is embeddable in R_s
   2) Each regular element of R in invertible in R_s
   3) Each element of R_s is of the form as^{-1}

(OR)

(b) Let M be a finitely generated free module over a commutative ring R. then prove that all bases of M are finite.

10. (a) State and prove the fundamental theorem of homomorphism for groups.

(OR)

(b) Prove that an ideal M of a commutative ring R with unity is maximal ideal $\Leftrightarrow \frac{R}{M}$ is a field.

*****

Code:1823/BL

Faculty of Sciences

M.Sc (Mathematics) I-Semester Backlog Examinations, Aug-2023

Paper- II: REAL ANALYSIS

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. If $\sum a_n$ converges then prove that $\lim_{n\to\infty} a_n = 0$.
2. If f(x)=3 ∀ rational 'x' and f(x)=4 ∀ rational x then prove that 'f' is not Riemann integrable on [a,b].
3. Show that $\sum_{n=1}^{\infty} \frac{1}{n^2x+1}$ converges uniformly on $[1,\infty]$.
4. If $A \in L(R^n, R^m)$ and $B \in L(R^m, R^k)$ then prove that $\| BA \| \le \| B \| . \| A \|$.
5. If $f.g \epsilon R(\alpha)$ on [a,b] such that $f(x) \le g(x) \forall x \in [a,b]$ then prove that $\int_a^b f d\alpha \le \int_a^b g d\alpha$.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) If f is a continuous function of a compact metric space X into a metric space Y, then prove that f is uniformly continuous on X.

(OR)

(b) If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X then prove that f(E) is a connected subset of Y.

7. (a) If P^* is the refinement of P, then prove that

$L(P,f,\propto) \le L(P^*,f,\propto)$ and $U(P^*,f,\propto) \le U(P,f,\propto)$.

(OR)

(b) Suppose $f:[a,b] \to R$ is continuous and $\propto:[a,b] \to R$ is a monotonic in increasing function then prove that $f \in R(\propto)$ on [a,b].

8. (a) Let $\propto$ be monotonically increasing on [a,b]. Suppose $\{f_n\}_{n=1}^{\infty}$ on is a sequence of real valued function defined on [a,b] such that $f_n \in R(\propto)$ on [a,b] for n=1,2,3..... and suppose $f_n \to f$ as $n \to \infty$ uniformly on [a,b]. then prove that $f_n \in R(\propto)$ on [a,b] and $\int_a^b f d\propto = \lim_{n\to\infty} \int_a^b f_n d\propto$.

(OR)

(b) State and prove Cauchy's criterion for uniformly convergence.

9. (a) Prove that a linear operator A on a finite dimensional vector space X is one - to - one if and only if the range of A is all of X.

(OR)

(b) Let Ω be the set of all invertible linear operators on R^n then prove Ω is an open subset of $L(R^n)$ and the mapping $A \to A^{-1}$ is continuous on Ω .

10. (a) If $f \in R$ on [a,b] and if there is a differentiable function F on [a,b] such that $F^1 = f$ then prove that $\int_a^b f(x)dx = F(b) - F(a)$.

(OR)

(b) Prove that the continuous image of a compact metric space is compact.

*****

Code: 1823/BL

Faculty of Sciences

M.Sc (Mathematics) I-Semester Backlog Examinations, Aug-2023

Paper-III: TOPOLOGY

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. State and prove Lindelof's theorem.
2. Prove that any closed subspace of a compact space is compact.
3. Show that in a Hausdroff space any point and disjoint compact subspace can be separated by open sets.
4. State and prove Tychnoff's theorem
5. Let (X,Y) be a topological space and $A \subseteq X$. Prove that $\bar{A} = A \cup D(A)$ where D(A) is the derived set of A.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) Prove that every second countable space is separable. Is the converse true? If Yes, prove. If No, give counter example

(OR)

(b) State and prove Kuratowski closure axioms.

7. (a) Define compact space. Prove that in a sequentially compact metric space, every open cover has a lebesgue number

(OR)

(b) Prove that a metric space is sequentially compact if and only if it has Bolzano-Weiestrass property.

8. (a) State and prove Urysohn imbedding theorem.

(OR)

(b) Let X be a T_1- space. Then prove that X is normal if and only if each neighborhood of a closed set F contains the closure of some neighborhood of F.

9. (a) Prove that the product of any non empty class of connected space is connected.

(OR)

(b) (i) Let X be a topological space and A be connected subspace of X. If B is a sub space of X such that $A \subseteq B \subseteq \bar{A}$ then prove that B is connected.

(ii) Prove that a topological space is disconnected if and only if there exists a continuous mapping of X onto the discrete two-point space{0,1}.

10. (a) Prove that a metric space is compact if and only if it is complete and totally bounded.

(OR)

(b) (i) Prove that a topological space X is a T_1- space if and only if each point set is a closed set.

(ii) A one-to-one continuous mapping of a compact space into a Hausdroff space is a homeomorphism.

*****

Code: 1824/BL

Faculty of Sciences

M.Sc (Mathematics) I-Semester Backlog Examinations, Aug-2023

Paper-IV: ELEMENTARY NUMBER THEORY

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each **(5x4=20 M)**

1. If $\frac{a}{bc}$ and $(a,b) = 1$, then prove that $\frac{a}{c}$
2. If P is a prime and $\frac{P}{ab}$ then $\frac{P}{a}$ or $\frac{P}{b}$
3. Define Euler totient function and find $\emptyset(3), \emptyset(4)$
4. If $a \equiv b$ (mod m) and d/m then $a \equiv b$(mod d).
5. If m=n (mod P) then prove that $\left(\frac{m}{p}\right) = \left(\frac{n}{p}\right)$

Section-B

II. Answer the following questions in not more than FOUR pages each **(5x10=50 M)**

6. (a) State and prove Division algorithm theorem.

(OR)

(b) Let a,b be two positive integers then prove that (a,b) [a,b] = a.b

7. (a) Prove that Dirichlet multiplication commutative and associative.

(OR)

(b) State and prove Mobius - inversion formula.

8. (a) If $a \equiv b$ (mod m) and $\propto \equiv \beta$ (mod m) then

   1) $ax + \propto y \equiv bx = \beta y$ (mod m)

   2) $a \propto \equiv b\beta$ (mod m)

   3) $a^n \equiv b^n$ (mod m) where n, x, y are positive integers.

(OR)

(b) If $P > 1$ integers with $(P-1)! \equiv -1$ (mod P) then P is prime

9. (a) For every odd prime, prove that $\left(\frac{-1}{p}\right) = (-1)^{\frac{p-1}{2}} = \begin{Bmatrix} 1 \; if \;\; P \equiv 1 \,(mod\ 4) \\ -1 \; if \;\; P \equiv 3 \,(mod\ 4) \end{Bmatrix}$

(OR)

(b) State and prove Gauss Lemma.

10. (a) Let d=(826,1890). Use Euclidean Algorithm and express d as a linear combination of 826, 1890.

(OR)

(b) Assume that (a,m)=1, then prove that the linear congruence $ax \equiv b\ (mod\ m)$ has unique solution.

*****

Code: 1825/BL

Faculty of Sciences

M.Sc (Mathematics) I-Semester Backlog Examinations, Aug-2023

Paper-V: MATHEMATICAL METHODS

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each **(5x4=20 M)**

1. Solve $z(z^2 + xy)(px - qy) = x^y$
2. Find the complete integral of the equation $p(1 + q) = qz$
3. Show that $\mathcal{J}_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{sinx}{x} - cosx\right)$
4. Use generating function of $P_n(x)$, prove that $P_n(1) = 1$
5. $(D^2 + DD^1 + 2D^{12})z = e^{x+y}$

Section-B

II. Answer the following questions in not more than FOUR pages each **(5x10=50 M)**

6. (a) Explain Charpit's method and hence using it to solve $p^2x + q^2y = Z$.

(OR)

(b) Find the eigen value and corresponding eigen function of the Strum- Liouville Boundary value problem $y^{11} + \lambda y = 0$; $y(0)$ *and* $y^1(L) = 0$.

7. (a) Reduce the equation $(n-1)^2 \frac{\partial^2 z}{\partial x^2} - (y)^2 \frac{\partial^2 z}{\partial y^2} = (ny)^{2n-1} \frac{\partial z}{\partial y}$ to canonical form, and find its general solution.

(OR)

(b) Obtain the general solution of heat equation by using the method of separation of variables.

8. (a) State and prove the orthogonality property of Bessels function.

(OR)

(b) Prove the following.

1) $\frac{d}{dx}[x^{-n}\mathcal{J}_n(x)] = -x^{-n}\mathcal{J}_{n+1}(x)$ **2)** $x\mathcal{J}_n^1(x) = -n\,\mathcal{J}_n(x) + x\,\mathcal{J}_{n-1}(x)$

9. (a) State and prove the Generating function for Laguerre polynomials.

(OR)

(b) Express $f(x) = x^3 - 2x^2 + 4x + 3$ in terms of Hermite polynomial

10. (a) Construct the Green's function from IVP $x^{11} = f(t), x(0) = 0, x(1) = 0$.

(OR)

(b)**(i)** Solve $\left(D^2 - DD^1 + 2D^{1^2}\right)z = X + Y$. **(ii)** Solve $\left(D^2 + D^{1^2}\right)z = cos\,mx\,cosny$

*****